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The Natural Coordinates on the Equiscalar Surface Undulating with Time*

Hideo KAWAI**

1. Introduction

HOLMBOE and his collaborators (1945) used the natural coordinates extensively in "Dynamical Meteorology", but their treatise was restricted within the horizontal flow in level surface, casting somewhat important role to geometrical intuition in the process of expressing equations by the natural coordinates. ARAKAWA (1951) tried to derive more consistently the equations of motion in the natuarl coordinates. However, they could not touch on the second derivative because their definition of the natural coordinates is mathematically obscure.

In a previous paper, the present author introduced analytically the natural coordinates basing on the extended directional differentiation and referring to the isobaric surfaces, and the geometry and kinematics of this coordinates were treated applying some part of them to the Kuroshio System (KAWAI, 1957). Though the vertical motion was taken into consideration and the second derivative was referred to, in order to simplify the mathematical expression there were adopted three assumptions upon the isobaric surfaces: the steady state of pressure field, the coincidence of isobaric surfaces with the surfaces of equal depth below the sea surface and the neglect of the curvature of isobaric surfaces. In short, this system was a natural coordinates on the isobaric surface, whose curvature is negligibly small. Thus, it is needful to combine more strictly the natural coordinates with isobaric surface or further with any equiscalar surface undulating with time.

The quasi-Lagrangian coordinates proposed by STARR (1945) were investigated in detail by ELIASSEN (1949), but the condition of the hydrostatic equilibrium was assumed and the vertical p-velocity divided by gravity force is substituted for the vertical velocity itself, neglecting the time change of the level of isobaric surfaces. The natural coordinates on an equiscalar surface in the present paper seems to be a combination of the natural coordinates with the quasi-Lagrangian coordinates, but it has not the character of the quasi-Lagrangian coordinates because the coordinate replaced for the vertical coordinate in the present system is not a scalar quantity but is a metric coordinate along the normal of the equiscalar surface.

In the course of deriving the present system it is not necessary to assume such conditions as assumed in HOLMBOE's natural coordinates and in ELIASSEN's quasi-Lagrangian coordinates, and the present system has the merit that we can derive, in its use, dynamical laws in complete form analytically without any aid of geometrical figures where finite difference is replaced for differential. As an application, the potential vorticity in the ocean is analytically derived (§ 7).

Since the natural coordinates on an equiscalar surface has the character of the so-called metric coordinates being essentially different from the curvilinear coordinates, the chief disadvantage in its use is that the respective coordinate is not independent of time and each other, which makes integration of the equation difficult, while the merit of the metric coordinates is that we can readily make practical calculations in its use. As another application using this merit, the orders of magnitude of terms appearing in the equations of motion expressed by the natural coordinates on the isobaric surface are estimated, and the equations are reduced to simpler forms dropping smaller terms.

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2. Orientation of the coordinate axes

Let the x and y axes take on a level surface, directed positive to the east and to the north respectively, and let the z axis take vertically, positive upward. When a current vector $\boldsymbol{V}$ is given at a point O, we can determine at O three unit vectors along the axes of the natural coordinates on an equiscalar surface as

$$\left.\begin{aligned} \boldsymbol{q} &\equiv \frac{\pm \operatorname{grad} f}{|\operatorname{grad} f|} \quad (\operatorname{grad} f \neq 0) \\ \boldsymbol{n} &\equiv \frac{\boldsymbol{q}\times\boldsymbol{V}}{|\boldsymbol{q}\times\boldsymbol{V}|} \quad (|\boldsymbol{q}\times\boldsymbol{V}| = |\boldsymbol{u}| \neq 0) \\ \boldsymbol{s} &\equiv \boldsymbol{n}\times\boldsymbol{q} \end{aligned}\right\}, \qquad (1)$$

where f indicates the scalar quantity whose equiscalar surface is designated as the coordinate surface. In the following, the equiscalar surface of f designated as the coordinate surface is called the *reference equiscalar surface* distinguishing it from the equiscalar surface of any other scalar quantity F, and the natural coordinates on an equiscalar surface of f are called the *natural f-coordinates* for simplicity. The double sign in front of grad f in the first equation of (1) has to be selected so that the unit vector $\boldsymbol{q}$ is directed upward.

The definition (1) is represented in words as follows; $\boldsymbol{s}$ is the unit vector of $\boldsymbol{u}$ which is the projected component of $\boldsymbol{V}$ normally upon the reference equiscalar surface of f passing through O, and $\boldsymbol{n}$ is the unit vector lying on the same equiscalar surface as $\boldsymbol{s}$ and is perpendicular, positive counter-clockwise, to $\boldsymbol{s}$, while $\boldsymbol{q}$ is the unit normal of the reference equiscalar surface, positive upward. The system of $\boldsymbol{s}$, $\boldsymbol{n}$ and $\boldsymbol{q}$ forms a right-handed orthogonal system locally at each point. If a vector field of current and a scalar field of f under consideration are given at an instant three-dimensionally, $\boldsymbol{s}$, $\boldsymbol{n}$ and $\boldsymbol{q}$ are determined at each point in so far as the current direction does not coincide with the normal of the equiscalar surface of f and grad f does not vanish.

Let $\boldsymbol{s}'$ and $\boldsymbol{n}'$ denote the vertically projected vectors of $\boldsymbol{s}$ and $\boldsymbol{n}$ upon a level surface respectively, then $\boldsymbol{s}'$ is not at right angles to $\boldsymbol{n}'$. Let θ_1 and θ_2 denote the angles of $\boldsymbol{s}'$ and $\boldsymbol{n}'$ made with the x and y axes, positive counter-clockwise, respectively. Let ε_1 and ε_2 denote the upward inclinations of $\boldsymbol{s}$ and $\boldsymbol{n}$ to a level surface respectively. Using these angles, from Fig. 1 the unit vectors along the axes of the natural f-coordinates $(\boldsymbol{s}, \boldsymbol{n}, \boldsymbol{q})$ are expressed by the unit vectors along the axes of the Cartesian coordinates $(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ as follows;

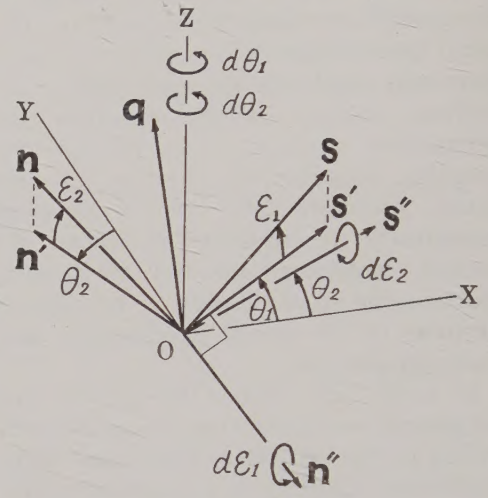

Fig. 1 Rotation of the coordinate system.

$$\left.\begin{aligned} \boldsymbol{s} &= T_{11}\boldsymbol{i} + T_{12}\boldsymbol{j} + T_{13}\boldsymbol{k} \\ \boldsymbol{n} &= T_{21}\boldsymbol{i} + T_{22}\boldsymbol{j} + T_{23}\boldsymbol{k} \\ \boldsymbol{q} &= \boldsymbol{s}\times\boldsymbol{n} = T_{31}\boldsymbol{i} + T_{32}\boldsymbol{j} + T_{33}\boldsymbol{k} \end{aligned}\right\}, \qquad (2)$$

where T_{ij} has the meaning:

$$\left.\begin{aligned} T_{11} &\equiv \cos\varepsilon_1\cos\theta_1 & T_{21} &\equiv -\cos\varepsilon_2\sin\theta_2 \\ T_{31} &\equiv \cos\varepsilon_1\sin\varepsilon_2\sin\theta_1 - \sin\varepsilon_1\cos\varepsilon_2\cos\theta_2 & & \\ T_{12} &\equiv \cos\varepsilon_1\sin\theta_1 & T_{22} &\equiv \cos\varepsilon_2\cos\theta_2 \\ T_{23} &\equiv -(\cos\varepsilon_1\sin\varepsilon_2\cos\theta_1 + \sin\varepsilon_1\cos\varepsilon_2\sin\theta_2) & & \\ T_{13} &\equiv \sin\varepsilon_1 & T_{23} &\equiv \sin\varepsilon_2 \\ T_{33} &\equiv \cos\varepsilon_1\cos\varepsilon_2\cos(\theta_2-\theta_1) & & \end{aligned}\right\}. \qquad (3)$$

Since any direction can be designated completely by three angles, there must be a relation among the four angles ε_1, ε_2, θ_1 and θ_2, and it is expressed from the perpendicularity of $\boldsymbol{n}$ to $\boldsymbol{s}$ by the formula:

$$\left.\begin{aligned} &\sin(\theta_2-\theta_1) = \tan\varepsilon_1\tan\varepsilon_2 \\ \text{or}\quad &T_{33} \equiv \cos\varepsilon_1\cos\varepsilon_2\cos(\theta_2-\theta_1) \\ &\quad = \sqrt{1-\sin^2\varepsilon_1-\sin^2\varepsilon_2} \end{aligned}\right\}. \qquad (4)$$

Differentiating (4), we have

$$T_{33}d\theta_1 + (\sin\varepsilon_2/\cos\varepsilon_1)\,d\varepsilon_1 = T_{33}d\theta_2 - (\sin\varepsilon_1/\cos\varepsilon_2)\,d\varepsilon_2. \qquad (4')$$

We can further verify from (4)

$$\left.\begin{aligned}\sum_{j=1}^{3}T_{ij}T_{kj}&=\delta_{ik}\ (^{i}_{k}=1,2,3)\\ \sum_{i=1}^{3}T_{ij}T_{ik}&=\delta_{jk}\ (^{j}_{k}=1,2,3)\end{aligned}\right\},\qquad(5)$$

where δ indicates the KRONECKER's symbol. Accordingly, we get from (2) and (5)

$$\left.\begin{aligned}\boldsymbol{i}&=T_{11}\boldsymbol{s}+T_{21}\boldsymbol{n}+T_{31}\boldsymbol{q}\\ \boldsymbol{j}&=T_{12}\boldsymbol{s}+T_{22}\boldsymbol{n}+T_{32}\boldsymbol{q}\\ \boldsymbol{k}&=T_{13}\boldsymbol{s}+T_{23}\boldsymbol{n}+T_{33}\boldsymbol{q}\end{aligned}\right\}.\qquad(6)$$

Using (2), (3), (4), (6) and (4′), the differentials of the unit vectors $\boldsymbol{s}$, $\boldsymbol{n}$ and $\boldsymbol{q}$ are expressed as

$$\left.\begin{aligned}d\boldsymbol{s}=\boldsymbol{n}d\theta_q-\boldsymbol{q}d\theta_n\qquad d\boldsymbol{n}=\boldsymbol{q}d\theta_s-\boldsymbol{s}d\theta_q\\ d\boldsymbol{q}=\boldsymbol{s}d\theta_n-\boldsymbol{n}d\theta_s\end{aligned}\right\},\qquad(7)$$

where

$$\left.\begin{aligned}d\theta_s&=\ T_{33}/\cos\varepsilon_2 d\varepsilon_2+\sin\varepsilon_1 d\theta_2\\ d\theta_n&=-T_{33}/\cos\varepsilon_1 d\varepsilon_1+\sin\varepsilon_2\, d\theta_1\\ d\theta_q&=T_{33}d\theta_1+\sin\varepsilon_2/\cos\varepsilon_1 d\varepsilon_1\\ &=T_{33}d\theta_2-\sin\varepsilon_1/\cos\varepsilon_2 d\varepsilon_2\end{aligned}\right\}.\qquad(8)$$

Now, we shall consider the geometrical meaning of $d\theta_s$, $d\theta_n$ and $d\theta_q$. The infinitesimal rotations $d\theta_1$ and $d\theta_2$ have the same unit axial vector $\boldsymbol{k}$, while $d\varepsilon_1$ and $d\varepsilon_2$ have the unit axial vectors $\boldsymbol{n}''$ and $\boldsymbol{s}''$ respectively. Here, $\boldsymbol{n}''$ and $\boldsymbol{s}''$ are normal to the $\boldsymbol{s}'-\boldsymbol{s}$ plane and to the $\boldsymbol{n}'-\boldsymbol{n}$ plane respectively (Fig. 1), and they are expressed by

$$\begin{aligned}\boldsymbol{n}''&=\boldsymbol{i}\sin\theta_1-\boldsymbol{j}\cos\theta_1\\ &=\boldsymbol{q}\sin\varepsilon_2/\cos\varepsilon_1-\boldsymbol{n}T_{33}/\cos\varepsilon_1\\ \boldsymbol{s}''&=\boldsymbol{i}\cos\theta_2+\boldsymbol{j}\sin\theta_2\\ &=-\boldsymbol{q}\sin\varepsilon_1/\cos\varepsilon_2+\boldsymbol{s}T_{33}/\cos\varepsilon_2.\end{aligned}$$

On the other hand, from the above equations, the last equation of (6) and (8), the infinitesimal rotations around $\boldsymbol{s}$, $\boldsymbol{n}$, and $\boldsymbol{q}$ axes are expressed by

$$\begin{aligned}\boldsymbol{s}\cdot(\boldsymbol{k}d\theta_2+\boldsymbol{s}''d\varepsilon_2)&=d\theta_s\\ \boldsymbol{n}\cdot(\boldsymbol{k}d\theta_1+\boldsymbol{n}''d\varepsilon_1)&=d\theta_n\\ \boldsymbol{q}\cdot(\boldsymbol{k}d\theta_1+\boldsymbol{n}''d\varepsilon_1)&=\boldsymbol{q}\cdot(\boldsymbol{k}d\theta_2+\boldsymbol{s}''d\varepsilon_2)=d\theta_q\end{aligned}$$

respectively. In words, $d\theta_s$, $d\theta_n$ and $d\theta_q$ signify the infinitesimal rotations of this coordinate system around the respective coordinate axes.

3. The directional differentiation along the axes of the natural f-coordinates

In this paragraph and the next, geometry and kinematics in the natural f-coordinates are introduced with the aid of the directional differentiation.

When a scalar function $F(x, y, z, t)$ is differentiable, the directional derivatives in the direction of $\boldsymbol{s}$, $\boldsymbol{n}$ and $\boldsymbol{q}$ are defined as follows:

$$\left.\begin{aligned}\frac{\partial F}{\partial s}&\equiv T_{11}\frac{\partial F}{\partial x}+T_{12}\frac{\partial F}{\partial y}+T_{13}\frac{\partial F}{\partial z}\\ \frac{\partial F}{\partial n}&\equiv T_{21}\frac{\partial F}{\partial x}+T_{22}\frac{\partial F}{\partial y}+T_{23}\frac{\partial F}{\partial z}\\ \frac{\partial F}{\partial q}&\equiv T_{31}\frac{\partial F}{\partial x}+T_{32}\frac{\partial F}{\partial y}+T_{33}\frac{\partial F}{\partial z}\end{aligned}\right\}.\qquad(9)$$

We have readily from (9) and (5)

$$\left.\begin{aligned}\frac{\partial F}{\partial x}&=T_{11}\frac{\partial F}{\partial s}+T_{21}\frac{\partial F}{\partial n}+T_{31}\frac{\partial F}{\partial q}\\ \frac{\partial F}{\partial y}&=T_{12}\frac{\partial F}{\partial s}+T_{22}\frac{\partial F}{\partial n}+T_{32}\frac{\partial F}{\partial q}\\ \frac{\partial F}{\partial z}&=T_{13}\frac{\partial F}{\partial s}+T_{23}\frac{\partial F}{\partial n}+T_{33}\frac{\partial F}{\partial q}\end{aligned}\right\}.\qquad(10)$$

Putting $F=\Omega$ (geopotential) in (9) and putting $F=f$ (reference scalar quantity) in the third equation of (10), we get from the conditions:

$$\frac{\partial\Omega}{\partial x}=\frac{\partial\Omega}{\partial y}=0\quad\text{and}\quad\frac{\partial f}{\partial s}=\frac{\partial f}{\partial n}=0$$

the formulas:

$$\left.\begin{aligned}\frac{\partial\Omega}{\partial s}&=g\sin\varepsilon_1 & \frac{\partial\Omega}{\partial n}&=g\sin\varepsilon_2\\ \frac{\partial\Omega}{\partial q}&=gT_{33} & \frac{\partial f}{\partial z}&=T_{33}\frac{\partial f}{\partial q}\end{aligned}\right\}.\qquad(11)$$

From the definition (9) it is verified that the directional differentiations of sum, product and quotient of two or more functions and a function of another function etc. can be calculated under the same rule as in the usual differentiation, even if θ_1, θ_2, ε_1 and ε_2 are variable with respect to x, y, z and t. However, when considering the derivatives of the second order, the difference between the usual differentiation and the variable directional differentiation revealed as shown below.

Differentiating (9) with respect to time t, we get from (10), (3), (8), (4) and (4′)

$$\left.\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial s}\right)-\frac{\partial}{\partial s}\left(\frac{\partial F}{\partial t}\right)&=\frac{\partial\theta_q}{\partial t}\frac{\partial F}{\partial n}-\frac{\partial\theta_n}{\partial t}\frac{\partial F}{\partial q}\\ \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial n}\right)-\frac{\partial}{\partial n}\left(\frac{\partial F}{\partial t}\right)&=\frac{\partial\theta_s}{\partial t}\frac{\partial F}{\partial q}-\frac{\partial\theta_q}{\partial t}\frac{\partial F}{\partial s}\\ \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial q}\right)-\frac{\partial}{\partial q}\left(\frac{\partial F}{\partial t}\right)&=\frac{\partial\theta_n}{\partial t}\frac{\partial F}{\partial s}-\frac{\partial\theta_s}{\partial t}\frac{\partial F}{\partial n}\end{aligned}\right\}.\qquad(12)$$

Putting $F=f$ in (12), we get

$$\left.\begin{aligned}\frac{\partial\theta_n}{\partial t}&=\frac{\dfrac{\partial}{\partial s}\left(\dfrac{\partial f}{\partial t}\right)}{\dfrac{\partial f}{\partial q}} & \frac{\partial\theta_s}{\partial t}&=\frac{-\dfrac{\partial}{\partial n}\left(\dfrac{\partial f}{\partial t}\right)}{\dfrac{\partial f}{\partial q}}\end{aligned}\right\}.\qquad(13)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial q}\right)=\frac{\partial}{\partial q}\left(\frac{\partial f}{\partial t}\right)\Bigg\}.$$

Substituting x, y, or z for t in (12) and (13), these equations are established, whereas s, n or q cannot be substituted for t in (12) and (13), and the corresponding equations have to be obtained newly from (9), (10), (3), (8), (4) and (4′) as below:

$$\left.\begin{aligned}
&\frac{\partial}{\partial n}\left(\frac{\partial F}{\partial s}\right)-\frac{\partial}{\partial s}\left(\frac{\partial F}{\partial n}\right)=\frac{\partial\theta_q}{\partial s}\frac{\partial F}{\partial s}\\
&\quad+\frac{\partial\theta_q}{\partial n}\frac{\partial F}{\partial n}-\left(\frac{\partial\theta_s}{\partial s}+\frac{\partial\theta_n}{\partial n}\right)\frac{\partial F}{\partial q}\\
&\frac{\partial}{\partial q}\left(\frac{\partial F}{\partial n}\right)-\frac{\partial}{\partial n}\left(\frac{\partial F}{\partial q}\right)=\frac{\partial\theta_s}{\partial n}\frac{\partial F}{\partial n}\\
&\quad+\frac{\partial\theta_s}{\partial q}\frac{\partial F}{\partial q}-\left(\frac{\partial\theta_n}{\partial n}+\frac{\partial\theta_q}{\partial q}\right)\frac{\partial F}{\partial s}\\
&\frac{\partial}{\partial s}\left(\frac{\partial F}{\partial q}\right)-\frac{\partial}{\partial q}\left(\frac{\partial F}{\partial s}\right)=\frac{\partial\theta_n}{\partial q}\frac{\partial F}{\partial q}\\
&\quad+\frac{\partial\theta_n}{\partial s}\frac{\partial F}{\partial s}-\left(\frac{\partial\theta_q}{\partial q}+\frac{\partial\theta_s}{\partial s}\right)\frac{\partial F}{\partial n}
\end{aligned}\right\}\quad(14)$$

Putting $F=f$ in (14), we get

$$\left.\begin{aligned}
&\frac{\partial\theta_s}{\partial s}+\frac{\partial\theta_n}{\partial n}=0\\
&\frac{\partial\theta_s}{\partial q}=\frac{-\frac{\partial}{\partial n}\left(\frac{\partial f}{\partial q}\right)}{\frac{\partial f}{\partial q}}\qquad\frac{\partial\theta_n}{\partial q}=\frac{\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial q}\right)}{\frac{\partial f}{\partial q}}
\end{aligned}\right\}\quad(15)$$

Substituting x, y and z respectively for t in (12), we get from (9) and (10)

$$\begin{aligned}
\Delta F=&\frac{\partial^2 F}{\partial s^2}+\frac{\partial^2 F}{\partial n^2}+\frac{\partial^2 F}{\partial q^2}+\left(\frac{\partial\theta_q}{\partial n}-\frac{\partial\theta_n}{\partial q}\right)\frac{\partial F}{\partial s}\\
&+\left(\frac{\partial\theta_s}{\partial q}-\frac{\partial\theta_q}{\partial s}\right)\frac{\partial F}{\partial n}+\left(\frac{\partial\theta_n}{\partial s}-\frac{\partial\theta_s}{\partial n}\right)\frac{\partial F}{\partial q}.
\end{aligned}\quad(16)$$

4. Vector operations

When a scalar function F is differentiable, we get from (6), (10) and (5)

$$\begin{aligned}
\mathrm{grad}\,F&\equiv\boldsymbol{i}\frac{\partial F}{\partial x}+\boldsymbol{j}\frac{\partial F}{\partial y}+\boldsymbol{k}\frac{\partial F}{\partial z}\\
&=\boldsymbol{s}\frac{\partial F}{\partial s}+\boldsymbol{n}\frac{\partial F}{\partial n}+\boldsymbol{q}\frac{\partial F}{\partial q},
\end{aligned}\quad(17)$$

so that the gradient on the equiscalar surface of f is defined as

$$\mathrm{grad}_f F\equiv\boldsymbol{s}\frac{\partial F}{\partial s}+\boldsymbol{n}\frac{\partial F}{\partial n}.\quad(17')$$

When a vector $\boldsymbol{A}$ is given, it is dissolved into three components in the two coordinates systems as

$$\boldsymbol{A}=\boldsymbol{i}A_x+\boldsymbol{j}A_y+\boldsymbol{k}A_z=\boldsymbol{s}A_s+\boldsymbol{n}A_n+\boldsymbol{q}A_q.\quad(18)$$

Considering (6) and (2), we have from (18)

$$\left.\begin{aligned}
A_s&=T_{11}A_x+T_{12}A_y+T_{13}A_z\\
A_n&=T_{21}A_x+T_{22}A_y+T_{23}A_z\\
A_q&=T_{31}A_x+T_{32}A_y+T_{33}A_2
\end{aligned}\right\}\quad(19)$$

and

$$\left.\begin{aligned}
A_x&=T_{11}A_s+T_{21}A_n+T_{31}A_q\\
A_y&=T_{12}A_s+T_{22}A_n+T_{32}A_q\\
A_z&=T_{13}A_s+T_{23}A_n+T_{33}A_q
\end{aligned}\right\}.\quad(20)$$

Proceeding now to the expression of div$\boldsymbol{A}$ by the natural f-coordinates, we have from (18)

$$\begin{aligned}
\mathrm{div}\,\boldsymbol{A}=&A_s\,\mathrm{div}\,\boldsymbol{s}+A_n\,\mathrm{div}\,\boldsymbol{n}+A_q\,\mathrm{div}\,\boldsymbol{q}\\
&+\frac{\partial A_s}{\partial s}+\frac{\partial A_n}{\partial n}+\frac{\partial A_q}{\partial q}.
\end{aligned}\quad(21)$$

Further, we get from (2), (10), (4) and (4′)

$$\left.\begin{aligned}
\mathrm{div}\,\boldsymbol{s}&=\frac{\partial\theta_q}{\partial n}-\frac{\partial\theta_n}{\partial q}\\
\mathrm{div}\,\boldsymbol{n}&=\frac{\partial\theta_s}{\partial q}-\frac{\partial\theta_q}{\partial s}\\
\mathrm{div}\,\boldsymbol{q}&=\frac{\partial\theta_n}{\partial s}-\frac{\partial\theta_s}{\partial n}
\end{aligned}\right\}\quad(22)$$

The divergence on the equiscalar surface of f is represented from (21) and (22) similarly as in the case neglecting the curvature of the equiscalar surface of f (Kawai, 1957) by the formula:

$$\mathrm{div}_f\boldsymbol{A}=\frac{\partial A_s}{\partial s}+\frac{\partial A_n}{\partial n}+A_s\frac{\partial\theta_q}{\partial n}-A_n\frac{\partial\theta_q}{\partial s}.\quad(21')$$

On the other hand, rot $\boldsymbol{A}$ is expressed from (18) and (17) by the formula:

$$\begin{aligned}
\mathrm{rot}\,\boldsymbol{A}=&A_s\,\mathrm{rot}\,\boldsymbol{s}+A_n\,\mathrm{rot}\,\boldsymbol{n}+A_q\,\mathrm{rot}\,\boldsymbol{q}\\
&+\boldsymbol{s}\left(\frac{\partial A_q}{\partial n}-\frac{\partial A_n}{\partial q}\right)+\boldsymbol{n}\left(\frac{\partial A_s}{\partial q}-\frac{\partial A_q}{\partial s}\right)\\
&+\boldsymbol{q}\left(\frac{\partial A_n}{\partial s}-\frac{\partial A_s}{\partial n}\right).
\end{aligned}\quad(23)$$

Further, we have from (7) and (22)

$$\left.\begin{aligned}
\mathrm{rot}\,\boldsymbol{s}&=-\boldsymbol{s}\left(\frac{\partial\theta_n}{\partial n}+\frac{\partial\theta_q}{\partial q}\right)+\boldsymbol{n}\frac{\partial\theta_n}{\partial s}+\boldsymbol{q}\frac{\partial\theta_q}{\partial s}\\
\mathrm{rot}\,\boldsymbol{n}&=\boldsymbol{s}\frac{\partial\theta_s}{\partial n}-\boldsymbol{n}\left(\frac{\partial\theta_q}{\partial q}+\frac{\partial\theta_s}{\partial s}\right)+\boldsymbol{q}\frac{\partial\theta_q}{\partial n}\\
\mathrm{rot}\,\boldsymbol{q}&=\boldsymbol{s}\frac{\partial\theta_s}{\partial q}+\boldsymbol{n}\frac{\partial\theta_n}{\partial q}
\end{aligned}\right\}\quad(24)$$

The rotation on the equiscalar surface of f is nothing but the $\boldsymbol{q}$-component of rot $\boldsymbol{A}$:

$$\begin{aligned}
\mathrm{rot}_f\boldsymbol{A}&\\
=&\frac{\partial A_n}{\partial s}-\frac{\partial A_s}{\partial n}+A_s\frac{\partial\theta_q}{\partial s}+A_n\frac{\partial\theta_q}{\partial n}.
\end{aligned}\quad(23')$$

Since the current vector $\boldsymbol{V}$ has not $\boldsymbol{n}$-component as shown in

$$\boldsymbol{V}=\boldsymbol{s}u+\boldsymbol{q}w,\quad(25)$$

the equations (21), (22), (21′), (23), (24) and

(23′) are, for the current vector, reduced to

$$\operatorname{div} \boldsymbol{V}=\frac{\partial u}{\partial s}+\frac{\partial w}{\partial q}+u\left(\frac{\partial \theta_q}{\partial n}-\frac{\partial \theta_n}{\partial q}\right)+w\left(\frac{\partial \theta_n}{\partial s}-\frac{\partial \theta_s}{\partial n}\right) \tag{26}$$

$$Q_f \equiv \operatorname{div}_f \boldsymbol{V}=\frac{\partial u}{\partial s}+u\frac{\partial \theta_q}{\partial n} \tag{26′}$$

$$\left.\begin{aligned} &\operatorname{rot} \boldsymbol{V}=\boldsymbol{s}\xi+\boldsymbol{n}\eta+\boldsymbol{q}\zeta \\ &\xi=\frac{\partial w}{\partial n}-u\left(\frac{\partial \theta_s}{\partial s}-\frac{\partial \theta_q}{\partial q}\right)+w\frac{\partial \theta_s}{\partial q} \\ &\eta=\frac{\partial u}{\partial q}-\frac{\partial w}{\partial s}+u\frac{\partial \theta_n}{\partial s}+w\frac{\partial \theta_n}{\partial q} \\ &\zeta \equiv \operatorname{rot}_f \boldsymbol{V}=-\frac{\partial u}{\partial n}+u\frac{\partial \theta_q}{\partial s} \end{aligned}\right\}. \tag{27}$$

5. Equation of motion and vorticity equation

In this paragraph the equation motion and the vorticicy equation are expressed in the frame of the natural f-coordinates. The equation of frictionless motion in atmosphere and ocean has the form:

$$\frac{\partial \boldsymbol{V}}{\partial t}+(\boldsymbol{W}\times \boldsymbol{V})+\alpha \operatorname{grad} p+\operatorname{grad}\left(\Omega+\frac{V^2}{2}\right)=0, \tag{28}$$

where

$$\boldsymbol{W}=\operatorname{rot} \boldsymbol{V}+2\boldsymbol{O}. \tag{29}$$

The symbols used here and in the following have the meaning.

$\boldsymbol{W}$: absolute vorticity vector
$\boldsymbol{O}$: angular velocity of the earths' rotation
α: specific volume ρ: density
φ: geographical latitude p: pressure
$\lambda, \bar{\lambda}$: Coriolis parameters

Considering (25) and (7), the term of acceleration in (28) is expressed by the natural f-coordinates as

$$\frac{\partial \boldsymbol{V}}{\partial t}=\boldsymbol{s}\left(\frac{\partial u}{\partial t}+w\frac{\partial \theta_n}{\partial t}\right)+\boldsymbol{n}\left(u\frac{\partial \theta_q}{\partial t}-w\frac{\partial \theta_s}{\partial t}\right)+\boldsymbol{q}\left(\frac{\partial w}{\partial t}-u\frac{\partial \theta_n}{\partial t}\right). \tag{30}$$

Expressing the angular velocity vector of the earth's rotatisn as

$$2\boldsymbol{O}=\boldsymbol{k}\lambda+\boldsymbol{j}\bar{\lambda}=\boldsymbol{s}2O_s+\boldsymbol{n}2O_n+\boldsymbol{q}2O_q$$
$$\lambda=2|\boldsymbol{O}|\sin\varphi \qquad \bar{\lambda}=2|\boldsymbol{O}|\cos\varphi,$$

the absolute vorticity vector is dissolved by (29) and (27) into three components:

$$\left.\begin{aligned} &\boldsymbol{W}=\boldsymbol{s}X+\boldsymbol{n}Y+\boldsymbol{q}Z \\ &X=\xi+2O_s, \quad 2O_s=T_{13}\lambda+T_{12}\bar{\lambda} \\ &Y=\eta+2O_n, \quad 2O_n=T_{23}\lambda+T_{22}\bar{\lambda} \\ &Z=\zeta+2O_q, \quad 2O_q=T_{33}\lambda+T_{32}\bar{\lambda} \end{aligned}\right\}. \tag{31}$$

Accordingly, the second term of (28) is expressed as

$$\boldsymbol{W}\times \boldsymbol{V}=\boldsymbol{s}wY+\boldsymbol{n}(uZ-wX)+\boldsymbol{q}uY. \tag{32}$$

Using (17), (30) and (32), (28) is dissolved into three components:

$$\left.\begin{aligned} &\frac{\partial u}{\partial t}+\frac{\partial}{\partial s}\left(\frac{V^2}{2}+\Omega\right)+\alpha\frac{\partial p}{\partial s}=-w\left(Y+\frac{\partial \theta_n}{\partial t}\right) \\ &\frac{\partial}{\partial n}\left(\frac{V^2}{2}+\Omega\right)+\alpha\frac{\partial p}{\partial n}=-u\left(Z+\frac{\partial \theta_q}{\partial t}\right)+w\left(X+\frac{\partial \theta_s}{\partial t}\right) \\ &\frac{\partial w}{\partial t}+\frac{\partial}{\partial q}\left(\frac{V^2}{2}+\Omega\right)+\alpha\frac{\partial p}{\partial q}=u\left(Y+\frac{\partial \theta_n}{\partial t}\right) \end{aligned}\right\}. \tag{33}$$

Here, it must not be overlooked that the respective component of the angular velocity due to the rotation of this coordinate system around the coordinate axis is added to the respective component of the absolute vorticity, though $\boldsymbol{s}$- and $\boldsymbol{n}$-components of them are numerically minute as shown in § 6.

Considering (27), (33) becomes

$$\left.\begin{aligned} &\frac{Du}{Dt}+\frac{\partial \Omega}{\partial s}+\alpha\frac{\partial p}{\partial s}=-w\left(\frac{D\theta_n}{Dt}+2O_n\right) \\ &\frac{\partial \Omega}{\partial n}+\alpha\frac{\partial p}{\partial n}=-u\left(\frac{D\theta_q}{Dt}+2O_q\right)+w\left(\frac{D\theta_s}{Dt}+2O_s\right) \\ &\frac{Dw}{Dt}+\frac{\partial \Omega}{\partial q}+\alpha\frac{\partial p}{\partial q}=u\left(\frac{D\theta_n}{Dt}+2O_n\right) \end{aligned}\right\}. \tag{34}$$

Operating rot to (28), we get

$$\frac{\partial \boldsymbol{W}}{\partial t}+(\boldsymbol{V}\cdot\operatorname{grad})\boldsymbol{W}-(\boldsymbol{W}\cdot\operatorname{grad})\boldsymbol{V}-\operatorname{grad} p\times\operatorname{grad}\alpha+\boldsymbol{W}\operatorname{div}\boldsymbol{V}=0. \tag{35}$$

This equation is dissolved by (17), (25), (31), (7) and (15) into three components, and the $\boldsymbol{q}$-component is expressed by

$$\frac{DZ}{Dt}-\left(\frac{\partial w}{\partial s}+\frac{\partial \theta_n}{\partial t}+w\frac{\partial \theta_n}{\partial q}\right)X-\left(\frac{\partial w}{\partial n}-\frac{\partial \theta_s}{\partial t}-w\frac{\partial \theta_s}{\partial q}\right)Y-\left(\frac{\partial w}{\partial q}-u\frac{\partial \theta_n}{\partial q}-\operatorname{div}\boldsymbol{V}\right)Z-\tilde{N}_q=0, \tag{36}$$

where

$$\tilde{N}_q=\frac{\partial p}{\partial s}\frac{\partial \alpha}{\partial n}-\frac{\partial p}{\partial n}\frac{\partial \alpha}{\partial s}.$$

Extending the vertical p-velocity, the vertical f-velocity is defined as

$$\omega \equiv \frac{Df}{Dt} = \frac{\partial f}{\partial t} + w\frac{\partial f}{\partial q},$$

and its directional derivative is shown from (13) and (15) by

$$\left.\begin{aligned} \frac{\partial \omega}{\partial s} &= \left(\frac{\partial \theta_n}{\partial t} + \frac{\partial w}{\partial s} + w\frac{\partial \theta_n}{\partial q}\right)\frac{\partial f}{\partial q} \\ \frac{\partial \omega}{\partial n} &= \left(-\frac{\partial \theta_s}{\partial t} + \frac{\partial w}{\partial n} - w\frac{\partial \theta_s}{\partial q}\right)\frac{\partial f}{\partial q} \\ \frac{\partial \omega}{\partial q} &= \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial q}\right) + \frac{\partial w}{\partial q}\frac{\partial f}{\partial q} + w\frac{\partial^2 f}{\partial q^2} \end{aligned}\right\} \quad (37)$$

Considering (37), the q-component of the vorticity equation (35) becomes

$$\frac{D}{Dt}\left(Z\frac{\partial f}{\partial q}\right) + Z\frac{\partial f}{\partial q}\operatorname{div} V = X\frac{\partial \omega}{\partial s} + Y\frac{\partial \omega}{\partial n} + Z\frac{\partial \omega}{\partial q} + \tilde{N}_q\frac{\partial f}{\partial q}. \quad (38)$$

This is a new expression of the vorticity equation.

6. Equations of motion in the natural coordinates on the isobaric surface

When f is equal to Ω and w vanishes in the natural f-coordinates, HOLMBOE's natural coordinates is derived. Putting $f=\sigma_t$ and $f=p$ in the natural f-coordinates, we get the natural σ_t-coordinates and the natural pressure coordinates respectively. Though the natural coordinates on the equiscalar surfaces of other physical quantities may be considered, they are not significant in the dynamical oceanography. In this paragraph, the orders of magnitude of terms in the equations of motion expressed by the natural pressure coordinates are estimated, and these equations are reduced to simpler forms, one of which comes to the equations derived in a previous paper.

Considering (11), (34) is reduced to

$$\frac{Du}{Dt} + g\sin\varepsilon_1 = -w\left(\frac{D\theta_n}{Dt} + 2O_n\right) \quad (34'.1)$$

$$g\sin\varepsilon_2 = -u\left(\frac{D\theta_q}{Dt} + 2O_q\right) + w\left(\frac{D\theta_s}{Dt} + 2O_s\right) \quad (34'.2)$$

$$\frac{Dw}{Dt} + gT_{33} = u\left(\frac{D\theta_n}{Dt} + 2O_n\right) - \alpha\frac{\partial p}{\partial q}, \quad (34'.3)$$

because $\frac{\partial p}{\partial s}$ and $\frac{\partial p}{\partial n}$ both vanish in the present coordinates.

We are now in a position to evaluate the orders of magnitude of all quantities appearing in the equations of motion expressed by the natural pressure coordinates in the ocean, after such method that CHARNEY (1948) tested for the atmospheric motion, adopting the characteristic magnitudes as shown below:

Radius of curvature of the isobaric stream line	$R\sim10^6$ cm
Length along the isobaric stream line	$S\sim10^7$ cm
Length across the isobaric stream lines	$N\sim10^6$ cm
Vertical thickness	$H\sim10^4$ cm
Current velocity	$V\sim10^2$ cm sec^{-1}
Wave velocity	$c\sim10^2$ cm sec^{-1}
Gravity acceleration	$g\sim10^3$ cm sec^{-2}
Coriolis parameters	$\lambda\sim\bar{\lambda}\sim10^{-4}$ sec^{-1}

Here, R corresponds to the radius of the eddy of intermediate size (SPILHAUS, 1940), S the mean length along the isobaric stream line between points at which velocity takes extreme values, N the width of the horizontal shear-zone, and H the thickness of the vertical shear-layer respectively. Roughly speaking, S is the mean distance between trough and wedge in the stream line pattern. Of course, the characteristic scales, S, N and H, defined for current velocity are different from the scales defined for other quantity, but the former is nearly of the same order with the latter.

The time dimension is characterized by V and c, which corresponds to the mean speed of propergation of the stream line pattern, neglecting such motions on a small scale as gravity waves ($c\sim10^4$ cm sec^{-1}) and tidal waves ($c\sim10^5$ cm sec^{-1}).

The orders of magnitude of the spatial derivatives can be estimated by replacing differentials by finite increment and expressing the incremental ratios in term of the characteristic magnitudes. The orders of magnitude of the time derivatives can be estimated by expressing as

$$\partial/\partial t \sim D/Dt \sim c\partial/\partial s \sim V\partial/\partial s.$$

Here, the symbol "$\sim$" denotes equality in orders of magnitude, while the symbol "$\approx$" appearing later on denotes the sign of equality in the approximate equation.

Since the inclination of the isobaric surface is numerically minute, the equations (3), (8) and (9) can be respectively approximated as

$$\left.\begin{aligned} &T_{11}\approx \cos\theta_1 \qquad T_{21}\approx -\sin\theta_2\\ &T_{31}\approx \varepsilon_2\sin\theta_1-\varepsilon_1\cos\theta_2\\ &T_{12}\approx \sin\theta_1 \qquad T_{22}\approx \cos\theta_2\\ &T_{32}\approx -(\varepsilon_2\cos\theta_1+\varepsilon_1\sin\theta_2)\\ &T_{13}\approx \varepsilon_1 \qquad T_{23}\approx \varepsilon_2\\ &T_{33}\approx 1-\varepsilon_1^2/2-\varepsilon_2^2/2 \end{aligned}\right\}, \quad (3A)$$

$$\left.\begin{aligned} &d\theta_s\approx d\varepsilon_2+\varepsilon_1 d\theta_2\\ &d\theta_n\approx -d\varepsilon_1+\varepsilon_2 d\theta_1\\ &d\theta_q\approx \varepsilon_2 d\varepsilon_1+d\theta_1\approx -\varepsilon_1 d\varepsilon_2+d\theta_2 \end{aligned}\right\} \quad (8A)$$

and

$$\left.\begin{aligned} \frac{\partial F}{\partial s}&\approx \cos\theta_1\frac{\partial F}{\partial x}+\sin\theta_1\frac{\partial F}{\partial y}+\varepsilon_1\frac{\partial F}{\partial z}\\ \frac{\partial F}{\partial n}&\approx -\sin\theta_2\frac{\partial F}{\partial x}+\cos\theta_2\frac{\partial F}{\partial y}+\varepsilon_2\frac{\partial F}{\partial z}\\ \frac{\partial F}{\partial q}&\approx (\varepsilon_2\sin\theta_1-\varepsilon_1\cos\theta_2)\frac{\partial F}{\partial x}\\ &\quad -(\varepsilon_2\cos\theta_1+\varepsilon_1\sin\theta_2)\frac{\partial F}{\partial y}+\frac{\partial F}{\partial z} \end{aligned}\right\}. \quad (9A)$$

Since ε_1 and ε_2 are minute, we can assume

$$\left.\begin{aligned} &|\varepsilon_1 F_h|\ll|\partial F/\partial z|\ll|F_h/\varepsilon_1|\\ &|\varepsilon_2 F_h|\ll|\partial F/\partial z|\ll|F_h/\varepsilon_2| \end{aligned}\right\}, \quad (39)$$

so we get from (9A)

$$\frac{\partial F}{\partial s}\leq F_h,\ \frac{\partial F}{\partial n}\leq F_h \text{ and } \frac{\partial F}{\partial q}\sim\frac{\partial F}{\partial z}, \quad (9A')$$

where F_h denotes the order of the greater magnitude of $\dfrac{\partial F}{\partial x}$ and $\dfrac{\partial F}{\partial y}$.

The assumptions:

$$\left.\begin{aligned} &|V|>|w|,\ \left|\frac{D\theta_q}{Dt}\right|>\left|\frac{D\theta_s}{Dt}\right|\\ &\text{and}\quad \left|\frac{D\theta_q}{Dt}\right|>\left|\frac{D\theta_n}{Dt}\right| \end{aligned}\right\} \quad (40)$$

are held for the current pattern of small scale. Considering

$$\frac{D\theta_q}{Dt}\sim\frac{\partial\theta_q}{\partial t}\sim\frac{V}{R}\sim 10^{-4}, \quad (41)$$

we get from (31), (3A) and (40)

$$\frac{D\theta_s}{Dt}+2O_s\sim\frac{D\theta_n}{Dt}+2O_n\sim\frac{D\theta_q}{Dt}+2O_q\sim 10^{-4}. \quad (42)$$

Further, considering that

$$\frac{Du}{Dt}\sim\frac{V^2}{S}\sim 10^{-3},$$

and assuming loosely for the present that $|w|<10\ \mathrm{cm\ sec^{-1}}$, we get from (34'.1), (34'.2) and (34'.3)

$$\varepsilon_1\sim 10^{-6},\ \varepsilon_2\sim 10^{-5} \text{ and } \frac{\partial p}{\partial q}\sim g\sim 10^3. \quad (43)$$

These orders of magnitude of ε_1 and ε_2 suppress the relative error of the approximate equations of (3A) less than 10^{-10}.

The angular velocity components of the rotation of coordinate axes can be estimated roughly as shown below. Considering

$$\frac{\partial\varepsilon_1}{\partial t}\sim\frac{V}{S}\varepsilon_1\sim 10^{-11},\quad \frac{\partial\varepsilon_2}{\partial t}\sim\frac{V}{S}\varepsilon_2\sim 10^{-10}$$

and $$\frac{\partial\theta_q}{\partial t}\approx\frac{\partial\theta_1}{\partial t}+\varepsilon_2\frac{\partial\varepsilon_1}{\partial t}\approx\frac{\partial\theta_2}{\partial t}-\varepsilon_1\frac{\partial\varepsilon_2}{\partial t},$$

which is derived from the third equation of (8A), we get from (41)

$$\frac{\partial\theta_q}{\partial t}\approx\frac{\partial\theta_1}{\partial t}\approx\frac{\partial\theta_2}{\partial t}\sim 10^{-4}. \quad (44.\ q)$$

Accordingly, we get from the first and the second equations of (8A)

$$\frac{\partial\theta_s}{\partial t}\approx\frac{\partial\varepsilon_2}{\partial t}+\varepsilon_1\frac{\partial\theta_2}{\partial t}\sim 10^{-10} \quad (44.\ s)$$

$$\frac{\partial\theta_n}{\partial t}\approx-\frac{\partial\varepsilon_1}{\partial t}+\varepsilon_2\frac{\partial\theta_1}{\partial t}\sim 10^{-9}. \quad (44.\ n)$$

The angular variation of coordinate axes along $\boldsymbol{s}$ and $\boldsymbol{n}$ can be estimated roughly as shown below.

Considering

$$\frac{\partial\varepsilon_1}{\partial s}\sim\frac{\varepsilon_1}{S}\sim 10^{-13},\quad \frac{\partial\varepsilon_2}{\partial s}\sim\frac{\varepsilon_2}{S}\sim 10^{-12},$$

$$\frac{\partial\theta_q}{\partial s}=\frac{1}{R}\sim 10^{-6}$$

and

$$\frac{\partial\theta_q}{\partial s}\approx\frac{\partial\theta_1}{\partial s}+\varepsilon_2\frac{\partial\varepsilon_1}{\partial s}\approx\frac{\partial\theta_2}{\partial s}-\varepsilon_1\frac{\partial\varepsilon_2}{\partial s},$$

which is derived from the third equation of (8A), we get

$$\frac{\partial\theta_q}{\partial s}\approx\frac{\partial\theta_1}{\partial s}\approx\frac{\partial\theta_2}{\partial s}\sim 10^{-6}. \quad (45.\ q)$$

Accordingly, we get from the first and the second equations of (8A)

$$\frac{\partial\theta_s}{\partial s}\approx\frac{\partial\varepsilon_2}{\partial s}+\varepsilon_1\frac{\partial\theta_2}{\partial s}\sim 10^{-12} \quad (45.\ s)$$

$$\frac{\partial\theta_n}{\partial s}\approx-\frac{\partial\varepsilon_1}{\partial s}+\varepsilon_2\frac{\partial\theta_1}{\partial s}\sim 10^{-11}. \quad (45.\ n)$$

Considering

$$\frac{\partial\varepsilon_1}{\partial n}\sim\frac{\varepsilon_1}{N}\sim 10^{-12},\quad \frac{\partial\varepsilon_2}{\partial n}\sim\frac{\varepsilon_2}{N}\sim 10^{-11}$$

and $$\frac{\partial\theta_n}{\partial n}=-\frac{\partial\theta_s}{\partial s}\sim 10^{-12}, \quad (46.\ n)$$

which is obtained from the first equation of 15) and (45.s), we get from the second equation of (8A)

$$\frac{\partial\theta_1}{\partial n}\approx\frac{1}{\varepsilon_2}\left(\frac{\partial\theta_n}{\partial n}+\frac{\partial\varepsilon_1}{\partial n}\right)\sim 10^{-7}.$$

Thus, we get from the third equation of (8A)

$$\frac{\partial\theta_q}{\partial n}\approx\frac{\partial\theta_1}{\partial n}\approx\frac{\partial\theta_2}{\partial n}\sim10^{-7}, \qquad (46.\,q)$$

and from the first equation of (8A)

$$\frac{\partial\theta_s}{\partial n}\approx\frac{\partial\varepsilon_2}{\partial n}+\varepsilon_1\frac{\partial\theta_2}{\partial n}\sim10^{-11}. \qquad (46.\,s)$$

The angular variation of coordinate axes along $\boldsymbol{q}$ cannot be estimated so easily as along $\boldsymbol{s}$ and $\boldsymbol{n}$. Differentiating (34′.3) with respect to n, and considering that the density difference between two points on both sides across the Kuroshio Front has the orders of magnitude $\Delta\rho\sim10^{-3}$, we get

$$\frac{\partial}{\partial n}\left(\frac{\partial p}{\partial q}\right)\sim g\frac{\partial\rho}{\partial n}\sim g\frac{\Delta\rho}{N}\sim10^{-6}.$$

Considering the third equation of (43) and the above equation, we get from the second equation of (15)

$$\frac{\partial\theta_s}{\partial q}=\frac{-\dfrac{\partial}{\partial n}\left(\dfrac{\partial p}{\partial q}\right)}{\dfrac{\partial p}{\partial q}}\sim10^{-9}. \qquad (47.\,s)$$

Seeing that

$$\frac{\partial\varepsilon_1}{\partial q}\sim\frac{\varepsilon_1}{H}\sim10^{-10} \quad\text{and}\quad \frac{\partial\varepsilon_2}{\partial q}\sim\frac{\varepsilon_2}{H}\sim10^{-9},$$

we get from the first equation of (8A) and (47.s)

$$\frac{\partial\theta_2}{\partial q}\approx\frac{1}{\varepsilon_1}\left(\frac{\partial\theta_s}{\partial q}-\frac{\partial\varepsilon_2}{\partial q}\right)\sim10^{-3},$$

so from that

$$\frac{\partial\theta_q}{\partial q}\approx\frac{\partial\theta_1}{\partial q}+\varepsilon_2\frac{\partial\varepsilon_1}{\partial q}\approx\frac{\partial\theta_2}{\partial q}-\varepsilon_1\frac{\partial\varepsilon_2}{\partial q}$$

we get

$$\frac{\partial\theta_q}{\partial q}\approx\frac{\partial\theta_1}{\partial q}\approx\frac{\partial\theta_2}{\partial q}\sim10^{-3}. \qquad (47.\,q)$$

We get, also, from the second equation of (8A) and (47.q)

$$\frac{\partial\theta_n}{\partial q}\approx-\frac{\partial\varepsilon_1}{\partial q}+\varepsilon_2\frac{\partial\theta_1}{\partial q}\sim10^{-8}. \qquad (47.\,n)$$

The order of magnitude of w can be estimated from the condition that div $\boldsymbol{V}$ vanishes in the incompressible fluid. Thus, we get from (26)

$$\frac{\partial w}{\partial q}+w\left(\frac{\partial\theta_n}{\partial s}-\frac{\partial\theta_s}{\partial n}\right)=-\frac{\partial u}{\partial s}-u\left(\frac{\partial\theta_q}{\partial n}-\frac{\partial\theta_n}{\partial q}\right)\sim10^{-5}.$$

Considering that

$$\frac{\partial w}{\partial q}\sim\frac{w}{H}\sim10^{-4}w\gg\left(\frac{\partial\theta_n}{\partial s}-\frac{\partial\theta_s}{\partial n}\right)w,$$

we get

$$\frac{\partial w}{\partial q}\approx-\frac{\partial u}{\partial s}\sim10^{-5}.$$

Accordingly, we obtain that $w\sim10^{-1}$.

From (44.q), (45.q), (46.q) and (47.q) the third equation of (8A) is reduced to

$$d\theta_q\approx d\theta_1\approx d\theta_2\approx d\theta,$$

so we can reject the suffixes 1, 2 and q hereafter, whereas the suffixes s and n of θ cannot be omitted.

Considering the orders of magnitude of ε_1 and ε_2 given in (43), the condition (39) becomes

$$10^{-5}F_h\ll\partial F/\partial z\ll10^5F_h. \qquad (48)$$

Then the equations (9A) become

$$\left.\begin{aligned}\frac{\partial F}{\partial s}&\approx\cos\theta\frac{\partial F}{\partial x}+\sin\theta\frac{\partial F}{\partial y}\\ \frac{\partial F}{\partial n}&\approx-\sin\theta\frac{\partial F}{\partial x}+\cos\theta\frac{\partial F}{\partial y}\\ \frac{\partial F}{\partial q}&\approx\frac{\partial F}{\partial z}\end{aligned}\right\}. \qquad (9B)$$

Corresponding to (48), we can assume that

$$10^{-5}F_v\ll\partial F/\partial q\ll10^5F_v, \qquad (49)$$

where F_v denotes the order of the smaller magnitude of $\dfrac{\partial F}{\partial s}$ and $\dfrac{\partial F}{\partial n}$. Then the equations (12) become from (40)

$$\left.\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\partial F}{\partial s}\right)-\frac{\partial}{\partial s}\left(\frac{\partial F}{\partial t}\right)&\sim10^{-4}\frac{\partial F}{\partial n}\\ \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial n}\right)-\frac{\partial}{\partial n}\left(\frac{\partial F}{\partial t}\right)&\sim10^{-4}\frac{\partial F}{\partial s}\\ \frac{\partial}{\partial t}\left(\frac{\partial F}{\partial q}\right)-\frac{\partial}{\partial q}\left(\frac{\partial F}{\partial s}\right)&\sim10^{-9}\frac{\partial F}{\partial s}\end{aligned}\right\}. \qquad (12A)$$

On the basis of (49), the equations (14) become

$$\left.\begin{aligned}\frac{\partial}{\partial n}\left(\frac{\partial F}{\partial s}\right)-\frac{\partial}{\partial s}\left(\frac{\partial F}{\partial n}\right)&\approx\frac{\partial\theta}{\partial s}\frac{\partial F}{\partial s}+\frac{\partial\theta}{\partial n}\frac{\partial F}{\partial n}\\ \frac{\partial}{\partial q}\left(\frac{\partial F}{\partial n}\right)-\frac{\partial}{\partial n}\left(\frac{\partial F}{\partial q}\right)&\approx-\frac{\partial\theta}{\partial q}\frac{\partial F}{\partial s}\\ \frac{\partial}{\partial s}\left(\frac{\partial F}{\partial q}\right)-\frac{\partial}{\partial q}\left(\frac{\partial F}{\partial s}\right)&\approx-\frac{\partial\theta}{\partial q}\frac{\partial F}{\partial n}\end{aligned}\right\}. \qquad (14A)$$

In regard to the vector operations we can approximate their expressions by the present natural coordinates as shown below.

Considering (47.n), (26) becomes

$$\text{div}\,\boldsymbol{V}\approx\frac{\partial u}{\partial s}+\frac{\partial w}{\partial q}+u\frac{\partial\theta}{\partial n}. \qquad (26A)$$

Also, the equations (27) and (31) become

$$\left.\begin{aligned}\xi &\approx \frac{\partial w}{\partial n}-u\frac{\partial\theta}{\partial q}\approx -u\frac{\partial\theta}{\partial q}\sim 10^{-1}\\ \eta &\approx \frac{\partial u}{\partial q}-\frac{\partial w}{\partial s}\approx \frac{\partial u}{\partial q}\sim 10^{-2}\\ \zeta &\approx -\frac{\partial u}{\partial n}+u\frac{\partial\theta}{\partial s}\sim 10^{-4}\end{aligned}\right\}\quad(27\mathrm{A})$$

and

$$\left.\begin{aligned}X &\approx \xi+\bar{\lambda}\sin\theta\sim 10^{-1}\\ Y &\approx \eta+\bar{\lambda}\cos\theta\sim 10^{-2}\\ Z &\approx \zeta+\lambda\sim 10^{-4}\end{aligned}\right\}\quad(31\mathrm{A})$$

respectively.

Accordingly, (34′) is reduced to

$$\left.\begin{aligned}\frac{\partial\Omega}{\partial s}+\frac{Du}{Dt} &\approx 0\\ \frac{\partial\Omega}{\partial n}+u\left(\frac{D\theta}{Dt}+\lambda\right) &\approx 0\\ \frac{\partial\Omega}{\partial z}+\alpha\frac{\partial p}{\partial z} &\approx 0\end{aligned}\right\}\quad(34'\mathrm{A})$$

Similarly, the equations of motion (33) are reduced to

$$\left.\begin{aligned}\frac{\partial u}{\partial t}+\frac{\partial}{\partial s}\left(\frac{V^2}{2}+\Omega\right) &\approx -wY\\ \frac{\partial}{\partial n}\left(\frac{V^2}{2}+\Omega\right) &\approx -u\left(Z+\frac{\partial\theta}{\partial t}\right)+wX\\ \frac{\partial w}{\partial t}+\frac{\partial}{\partial z}\left(\frac{V^2}{2}+\Omega\right) &\approx uY-\alpha\frac{\partial p}{\partial z}\end{aligned}\right\}\quad(33\mathrm{A})$$

Since the equations (9B), (12A), (14A), (26A), (27A), (31A) and (33A) derived above have the same forms with the equations derived under some assumptions upon the isobaric surface in a previous paper (Kawai, 1957), we can conclude that the effects of the motion and the curvature of isobaric surfaces are negligibly small in the motion of the scale mentioned at the beginning of this paragraph.

7. Various forms of the vorticity equation

The vorticity equation (38) is the most original form, for the various expressions of the vorticity equation may be derived from it when giving various scalar quantities to f in (38). The common assumptions made in deriving the following equations are that the motion is frictionless and that the fluid is incompressible.

(i) *Referred to level surface*

Putting $f=\Omega=gz$, the vertical f-velocity becomes $\omega=D\Omega/Dt=gw$, and we have $\partial f/\partial q=g$. Considering the above equations and the incompressibility of the fluid, the vorticity equation (38) is reduced to the usual form:

$$\frac{DZ}{Dt}=(\boldsymbol{W}\cdot\operatorname{grad}w)+\tilde{N}_z. \qquad (38.1)$$

(ii) *Referred to isobaric surface*

Putting $f=p$, $\tilde{N}_p$ vanishes. Accordingly, the vorticity equation (38) is reduced to

$$\frac{D}{Dt}\left(Z\frac{\partial p}{\partial q}\right)=(\boldsymbol{W}\cdot\operatorname{grad}\omega). \qquad (38.2)$$

If the motion is isobaric, the vertical p-velocity $\frac{Dp}{Dt}$ vanishes, so that $Z\frac{\partial p}{\partial q}$ is conserved.

(iii) *Referred to the equiscalar surface of conservative quantity f_c*

In this case the vertical f-velocity vanishes, so that the vorticity equation (38) is reduced to

$$\frac{D}{Dt}\left(Z\frac{\partial f_c}{\partial q}\right)=\tilde{N}_{fc}\frac{\partial f_c}{\partial q}. \qquad (38.3)$$

(iv) *Referred to isentropic surface*

In the ocean, the σ_t surfaces can be considered as being nearly equivalent to the isentropic surfaces in a dry atmosphere. Putting $f=\sigma_t$ and considering the incompressibility of the water, the vertical σ_t-velocity $\frac{D\sigma_t}{Dt}$ and the solenoidal term $\tilde{N}\sigma_t$ both vanish. Accordingly, the vorticity equation (38) is reduced to

$$\frac{D}{Dt}\left(Z\frac{\partial\sigma_t}{\partial q}\right)=0. \qquad (38.4)$$

This equation corresponds to the conservation equation of the potential vorticity in a dry atmosphere derived by Charney (1948), supposing a cylinder with sides perpendicular to two isentropic surfaces infinitesimally close together and with infinitesimal cross-section. A clear physical insight contributed much to his success, whereas the vorticity equation (38.4) could be derived analytically.

Thus, the potential vorticity in the ocean may be defined, most reasonablly, as $Z\frac{\partial\sigma_t}{\partial q}$

or Z grad σ_t, where $\frac{\partial}{\partial q}$ signifies the directional differentiation along the upward normal of the σ_t surface, and Z the vorticity component normal to the σ_t surface.

Numerically, $\frac{\partial}{\partial q}$ is nearly equal to $-\frac{\partial}{\partial z}$ for the σ_t surface, so that $10^{-3}\frac{\partial \sigma_t}{\partial q}$ is equivalent to the vertical stability in the upper layer of the ocean (HESSELBELG and SVERDRUP, 1915). On the other hand Z indicates the horizontal stability in zonal or circular flow. Accordingly, the equation (38.4) signifies that the product of the vertical stability and the horizontal stability is conserved for the isentropic motion in the incompressible water, assuming that the friction is very small. In the water mass stable horizontally and vertically ($\frac{\partial \sigma_t}{\partial q}>0$, $Z>0$), if the vertical stability of the water mass increases during motion, its horizontal stability decreases; while if the former decreases, the latter increases, namely—the two stabilities are compremmentary each other.

8. Concluded summary

The geometry and kinematics in the natural coordinates on an equiscalar surface, which undulates with time, were derived basing on the extended directional differentiation, and the equations of motion and the vorticity equation were represented in the frame of this coordinates. The merit in the use of the present coordinates is that we can analytically derive the dynamical laws in complete form, affording a more natural view in the transformation of equations. For instance, the law of the parallel solenoidal field (KAWAI, 1955) and the law of the conservation of the potential vorticity can be derived in a more fundamental form. As another appication it was verified that we can numerically neglect the effects of the undulation of isobaric surfaces in the equations of motion referred to isobaric surfaces. Thus, the equations derived in part I of the previous paper (KAWAI, 1957), neglecting the curvature of isobaric surfaces, can be established as they were, in so far as the characteristic scale of the phenomenon under consideration is large to some extent. However, when the phenomenon of small scale is considered, the present coordinate system will be effective.

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On the Fluctuation of the Kuroshio South of Honshū (4) (The Influence of the Oceanographic Conditions upon the Monthly Mean Sea Level)*

Shigeo MORIYASU**

1. Introduction

Only the tidal observation in the oceanographic observations in the broad sense gives us the continuous record. So, the tidal record is very significant and available for various purposes. There are many tidal stations along the south coast of Honshū, where continuous and accurate records are accumulated for years. These data have been used in the study of the storm surge and Tsunami by many oceanographers. However, a few works have been done concerning the influence of the oceanographic conditions upon the sea level in the considered area. The influence is expected to be significant, because the Kuroshio flows comparatively near the south coast of Honshū and its striking fluctuation has been found since the oceanographic observations were frequently carried out.

As the sea level contains many factors, they must be taken into account. According to MONTGOMERY (1937-8), the possible causes of long period variations in the sea level reduced to normal pressure can be conveniently divided into eight causes, in which dynamic effect due to variations in gradient currents and thermal effect are of most importance for Miami while Windstau (wind effect) also exerts a large influence at Charleston. NOMITSU and OKAMOTO (1927) discussed the annual variation of the mean sea level along the Japan coast and concluded that the most effective causes were the influence of the density change due to inequalities in temperature and salinity and that the effect of atmospheric pressure came second and that the effect of Coriolis force was not so small for the Pacific coast. The first and the third effect seems to be equivalent to one which was called thermal and dynamic effect, respectively, by MONTGOMERY. Furthermore, MIYAZAKI (1953) explained the annual mean sea level, from which the progressing trend was eliminated, as the effect of water density, atmospheric pressure and ocean current.

The relation between the dynamic height of the sea surface and the sea level at the adjacent tidal station was studied in some areas. MONTGOMERY (loc. cit.) found that variations in the dynamic height of the sea surface in the slope water on the left side of the Gulf Stream agreed with monthly sea level at Charleston and Miami. But, MONTGOMERY (1941) concluded that the values of sea level computed for the northern end of the *Atlantis* section across the Florida Current between Key West and Habana showed no appreciable correlation with sea level observed at the adjacent tidal station at Key West. On the variations in sea level along the west coast of the United States, it was introduced from the weekly sea level by LAFOND (1939) that they could be accounted for by the variations in the density of the water above 500 db. Thus, the close relation is not always found between them. Furthermore, ISELIN (1940) and MONTGOMERY (loc. cit.) studied the transport of the Gulf Stream and the Florida Current, respectively, in connection with the tidal data. STOMMEL (1953) discussed the seasonal fluctuations of the monthly sea level around the Florida Peninsula and explained a drop in the level of the free surface from Key West to Miami by applying the vorticity theorem to the Florida Current. Besides, the influence of the mete-

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** Kobe Marine Observatory

orological conditions over the North Pacific upon the sea level along the Pacific Coast of the United States was considered by JACOBS (1939).

These works are concerning the variation of long period, but not of short period. As to the latter, there are some works, for instance, by SHOJI (1954) and GROVES (1957).

In this paper, the relation between the monthly mean sea level at four tidal stations around the Kii Peninsula (Fig. 1) and the oceanographic conditions should be discussed. Here, it is assumed that the sea level rises (falls) by 1 cm corresponding to the fall (rise) of atmospheric pressure by 1 mb, and the effect due to the atmospheric pressure is eliminated statically by reducing the mean sea level to the standard atmospheric pressure. And the value of sea level is referred to the mean sea level of Tokyo Bay for simplicity in the comparison with that at other tidal station. The vertical land deformation is thought to be negligibly small during such a short period as treated in this paper. The oceanographic conditions are determined on the basis of the data reported by Japan Meteorological Agency (or former Central Meteorological Observatory), Kobe Marine Observatory and Maritime Safety Agency.

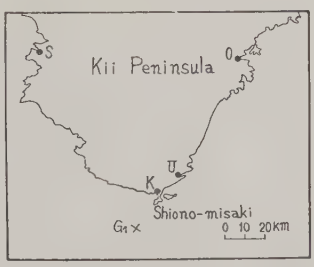

Fig. 1 The position of the tidal station. O: Owase, U: Uragami, K: Kushimoto, S: Shimotsu.

2. Monthly mean sea level at Kushimoto Tidal Station

Fig. 2 shows the monthly mean sea level at Kushimoto from 1951 to 1956, reduced to the standard atmospheric pressure by using the monthly mean atmospheric pressure at Shiono-misaki Weather Station. It may be considered that, during the period, there is not such a striking vertical land deformation as affects the sea level. The notable features of the fluctuation in the monthly mean sea level are as follows:

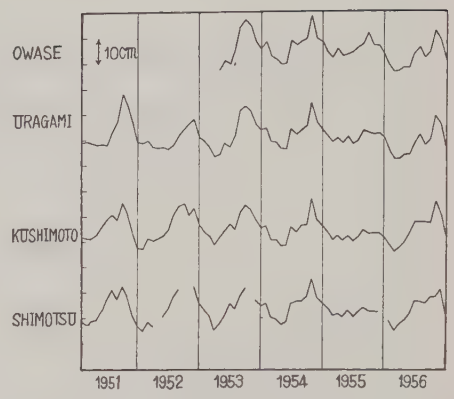

Fig. 2 The fluctuation of the monthly mean sea level reduced to the standard atmospheric pressure.

(1) the maximum value occurs in summer or autumn, sometimes accompanied with two peaks,

(2) the minimum value occurs in winter or spring,

(3) the annual variation in 1955 is considerably different from those in other years, namely, the annual variation is monotonous and there is the maximum in August which is lower by about 20 cm than that in other years.

The occurrence of the maximum in summer or autumn is due to the decrease of density caused by the rise of water temperature and the fall of chlorinity, while the occurrence of the minimum in winter is due to the increase of density caused by the fall of water temperature. Moreover, it is significant that the annual variation of the monthly mean sea level is varied from year to year. This seems to be in close connection both with the oceanographic conditions and with the influence of density (NOMITSU and OKAMOTO, loc. cit.) or the thermal effect (MONTGOMERY, loc. cit.).

MONTGOMERY (loc. cit.) pointed out that variations in the dynamic height of the sea surface were found to agree with monthly mean sea level reduced to normal atmospheric pressure. Here, the relationship between the monthly mean sea level at Kushimoto and

the fluctuation of the Kuroshio should be considered from the same point of view as his. Kushimoto Tidal Station is located in a small inlet, but it is considered to show the undeformed influence of the open sea because the station in good exposure and the depth is larger than 1000 m where it is about 10 miles apart from the station. Thus, the close relation may be expected between them. Fig. 3 shows the scatter diagram between the monthly mean sea level at Kushimoto and the dynamic height anomaly above the 800 db surface at oceanographic station G_1 (33°20′N, 135° 40′E) shown in Fig. 1. Since the monthly mean sea level contains many effects and the dynamic height anomaly is computed from the only one observation in the month, it is natural that the scatter in the diagram is somewhat remarkable. However, there may be a tendency that the larger the dynamic height anomaly is, the higher the monthly mean sea level is. The correlation coefficient is 0.830 when 16 examples from May, 1954 to November, 1956 are used. In this case, the significance point for $P=0.1\%$ is 0.7420 (for $N=16$) and so the relationship between them is concluded to be significant. Accordingly the sea level seems to be dependent upon the fluctuation of the Kuroshio because the dynamic height anomaly at G_1 represents the fluctuation of the Kuroshio. The striking features of the fluctuations in the oceanographic conditions during the considered period are as follows:

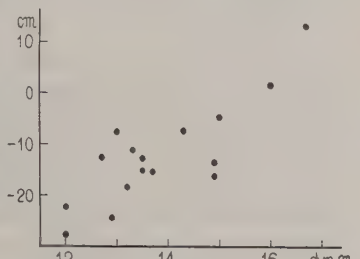

Fig. 3 The relation of the monthly mean sea level at Kushimoto to the dynamic height anomaly above the 800 db surface at G_1.

until the end of 1955—the axis of the Kuroshio was located far southward off Shiono-misaki and the cold water region was large off Enshū-nada where the meandering the Kuroshio was strinking;

in 1956—the cold water region was small and there was no strinking meandering of the Kuroshio which ran near the coast off Shiono-misaki.

In a word, the oceanographic conditions fluctuated remarkably in the considered period.

From the dependence of the sea level at Kushimoto upon the dynamic height anomaly at G_1, it is expected that the sea level is also related with the position of the axis of the Kuroshio. In Fig. 4 is shown the relation between the departure of the monthly mean sea level at Kushimoto and the position of the axis of the Kuroshio on the meridian of 135°40′E. Here, the position of the axis is determined from the geopotential topography at the surface referred to the 800 db surface which is based on the only one observation in the month and the departure is computed from the mean value of the monthly mean sea level in each month from 1951 to 1956. Using of the departure of the monthly mean sea level means to eliminate the effect owing to the annual variation of the density to a certain extent. From this figure, though the relation appears to be at random, the following tendency may be found: negative departures become larger as the axis goes away from the coast, while positive departures become larger as the axis is situated far offshore (about south of 32°20′N). Now, let $F(y)$ be the elevation of the sea surface in dynamic meter when the axis is situated at $y=y_0$. Here, y is the distance measured northward on the meridian. Then we can represent

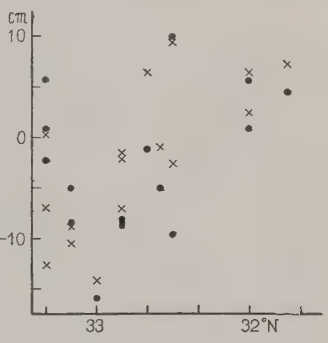

Fig. 4 The relation between the departure of the monthly mean sea level and the position of the current axis on the meridian of 135°40′E. ●: Kushimoto, ×: Uragami.

the elevation of the sea surface, $F'(y)$, when the axis is situated at $y=y_0+\Delta y_0$ as follows:

$$F'(y)=F(y-\Delta y_0)=F(y)-\frac{\partial F}{\partial y}\Delta y_0$$

$$=F(y)+\frac{f}{10}u\Delta y_0,$$

where it is assumed that the shape of the surface is not varied. Accordingly, the difference between them is

$$\Delta F=F'(y)-F(y)=\frac{f}{10}u\Delta y_0.$$

This is represented in meter as follows:—

$$\Delta h=\frac{f}{9.8}u\Delta y_0.$$

It follows from the above relation that as the axis moves northwards (southwards), the level rises in the region of eastward (westward) current and that the amount is proportional to the current velocity (u) at the station and the difference (Δy_0) in the position of the axis. The relation for $u>0$ and $\Delta y_0>0$ is shown in Fig. 5. As to the sea south of Shiono-misaki, the sea level becomes higher with the approaching of the axis to the coast when the eastward current exists, and with the going away of the axis from the coast when the westward current exists under the influence of the cold water region. This is very natural from the geopotential topography, but very interesting. Consequently, the rise of the sea level at Kushimoto does not always mean that the Kuroshio approaches the coast.

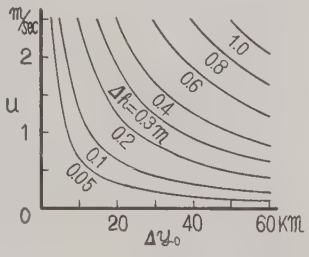

Fig. 5 The influence of the movement of the current axis upon the sea level for $u>0$ and $\Delta y_0>0$.

The monthly mean sea level at Shimotsu (Fig. 2) indicates the similar fluctuation as that at Kushimoto, and the correlation coefficient is 0.847 between the departures of the monthly mean sea level at both tidal stations from 1951 to 1956. As the significance point for $P=0.1\%$ is 0.3938 (for $N=67$), it is indicated that the influence upon the sea level at Kushimoto appears at Shimotsu without being modified. In other words, the fluctuation of the Kuroshio and the cold water region affects the sea level in the Kii-suido, too. Here, the pressure effect is eliminated by using the monthly mean atmospheric pressure at Wakayama Weather Station.

3. Monthly mean sea level at Uragami Tidal Station

The monthly mean sea level reduced to the standard atmospheric pressure is shown in Fig. 2, where the monthly mean atmospheric pressure at Shiono-misaki Weather Station is used. As Uragami Tidal Station is apart about 15 km from Kushimoto Tidal Station, it is expected that the sea levels at both stations fluctuate in similar manner. However, sometimes considerable differences can be found between them from this figure, for instance, in 1952. On the other hand, the fluctuation at Uragami in 1954 and 1955 is similar to that at Kushimoto.

The sea level at Uragami is considered to be dependent upon the oceanographic conditions as that at Kushimoto. The departure of the monthly mean sea level is positive in 1954 and in the first half of 1955 when the cold water region was large, while mostly negative in 1956 when the Kuroshio ran very near the coast off Shiono-misaki and the cold water region was small. From Fig. 4 indicating the relation between the departure of the monthly mean sea level at Uragami and the position of the axis, it is found that the more southward the axis exists, the larger the positive departure is. Namely, the rise of the sea level owing to the influence of the Kuroshio does not occur when the axis is situated near the coast, but the rise of the sea level under the influence of the counter-clockwise eddy accompanied by the cold water region is found when the axis is situated far southward. Hence, it may be thought that the difference in the sea level between both stations is large in the former case, while it is small in the latter case. The qualitative expression noted above is very rough, but it may be concluded that the most effec-

tive cause of the difference in the sea level seems to be in close relation to the oceanographic conditions.

Now, the monthly mean sea level at Uragami should be compared with that at Owase instead of that at Kushimoto in the preceding paragraph. The monthly mean sea level at Owase is reduced to the standard atmospheric pressure by using the monthly mean atmospheric pressure at Owase Weather Station. Fig. 2 shows the monthly mean sea level at Owase. From this, it seems to be nearly determined by the oceanographic conditions in Enshū-nada and the following features are seen: when the cold water region and so the counter-clockwise eddy is large, the sea level is high; in the contrary case, the sea level is low. These are also ascertained by the departure of the monthly mean sea level. The sea level at Uragami, shown above, has the nearly same tendency as this and is considered to be in close connection with that at Owase. The correlation coefficient between the departures of the monthly mean sea level at both stations from 1954 to 1956 is 0.848. As the significance point for $P=0.1\%$ (for $N=36$) is 0.5259, it is concluded that there is a significant relationship between them. Consequently, the sea level at Uragami may be regarded as dependent upon the oceanographic conditions in Enshū-nada. It is very interesting that the sea level at Uragami is more connected with that at Owase than that at Kushimoto, although Uragami is more distant from Owase than Kushimoto.

4. The difference in the sea level between Kushimoto and Uragami

From the above expressions, the difference in the monthly mean sea level between both stations is expected to fluctuate remarkably according to the oceanographic conditions, although the distance between them is not large. The difference (Kushimoto minus Uragami) shown in Fig. 6 is nearly constant below 5cm in the second half of 1953, and in 1954 and 1955, and is much varied and sometimes exceeds 20 cm in other period. This has been pointed out by Yoshizawa (1957).

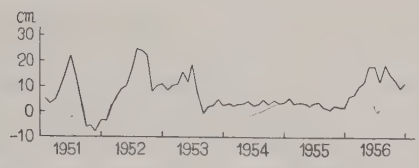

Fig. 6 The fluctuation of the difference in the monthly mean sea level between Kushimoto and Uragami (Kushimoto minus Uragami).

The difference shown in the figure is the absolute value because the sea level is the value above the mean sea level of Tokyo Bay. The fluctuation of the difference seems to be striking when the cold water region is small, while to be nearly constant when it is large. The fluctuation of the cold water region may be represented by the position of the axis on the ground that, in general, the larger the cold water region is, the more southward the axis is situated, and *vice versa*. Accordingly, the relation between the difference and the position of the axis on the meridian of 135°40′E is shown in Fig. 7 in order to deduce the influence of the cold water region upon the difference in the sea level. From the figure, the difference is below 5 cm when the axis is far apart from the coast, while it is large and varied and sometimes exceeds 20 cm when the axis is north of 33°N. This is the general feature and there are a few exceptions. Thus, there is no well-defined quantitative relationship between them, but the qualitative one may be seen, which is divided into two parts by the critical position of the axis, about 33°N. At the end of 1951, the difference is negative. Such negative value has rarely been found. The reason why the negative difference occurs can

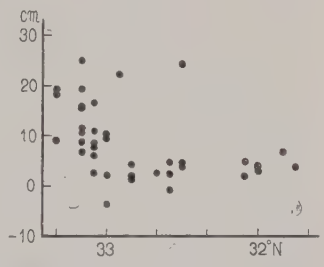

Fig. 7 The relation between the difference in the monthly mean sea level (Kushimoto minus Uragami) and the position of the current axis on the meridian of 135° 40′E.

not be made clear because of the scatiness in the oceanographic observations. Nevertheless, itis known only that the axis was about 50 miles apart from Shiono-misaki in December, 1951.

The vertical land deformation is considered as one of the causes of the fluctuation in the difference in the sea level between two stations. However, we can not confirm that there is a striking vertical land deformation in a small region during such a short period. Consequently, we need not attach weight to the vertical land deformation in this problem.

Besides, we may take up the differences in the various elements which affect the sea level. The inequality of the thermal effect causes the increases of the difference, for instance in summer of 1951, 1952 and 1956. But, this does not affect the essential feature of the fluctuation in difference in the sea level. Though the influence of the Windstau is also regarded as one cause, it seems not to be of importance. Thus, the oceanographic conditions, that is, the cold water region and the Kuroshio, are most important.

5. Two typical examples

It is stated in the preceding sections that the sea level, in which the pressure effect is not included, fluctuates mainly according to the oceanographic conditions. In this section, we will show two examples each of which indicates the distribution of the sea level peculiar to the oceanographic conditions.

The one example is in February, 1955 and the other is in August, 1956. In the former case, the cold water region was large and the axis of the Kuroshio was far off-shore south of Shiono-misaki, while in the latter case, the cold water region was small and the axis was near the coast. In a word, the oceanographic conditions were completely contrary in each case. In Table 1, the departures of the monthly mean sea level at Owase, Uragami, Kushimoto and Shimotsu, and the difference in the monthly mean sea level between Uragami and Kushimoto are shown. In the former case, the departure at each station has large positive value and the difference in the sea level between Uragami and Kushimoto is small, that is, these represent the feature corresponding to the condition that the axis is far apart from the coast. On the other hand, in the latter case, the departure at each station has negative value the absolute value of which is large at Owase and Uragami, and the difference in the sea level between Uragami and Kushimoto is very large, that is, these represent the feature peculiar to the condition that the axis is near the coast.

6. The results of the harmonic analysis of the monthly mean sea level

The annual variation of the monthly mean sea level is not constant in each year. This will be made clear also by the harmonic analysis. In Table 2, the results for each year are shown, where

$$h=h_0+h_1\cos(\sigma t-\mu_1)+h_2\cos(2\sigma t-\mu_2)$$

and $\sigma=2\pi/12$ months. The constants obtained by the process do not represent the exact values of S_a and S_{sa} as pointed out by MIYAZAKI (1955). However, their departures from S_a and S_{sa} are not so large that the feature peculiar to the year is distorted. As the effect of the atmospheric pressure is eliminated from the monthly mean sea level, the constants are somewhat different from those when it is not eliminated, because the monthly mean pressure shows also an annual variation the phase of which differs from that of the annual varia-

Table 1. The various elements in two typical cases. Δh: the departure of the monthly mean sea level, ΔH: the difference in the monthly mean sea level between Kushimoto and Uragami, A: the position of the current axis on the meridian of 135°40′E.

	Δh (cm)				ΔH (cm)	A
	Owase	Uragami	Kushimoto	Shimotsu		
February, 1955	3.4	2.4	0.8	4.6	3.2	32° 00′ N
August, 1956	−7.7	−12.7	−2.3	−2.7	19.2	33°-20′ N

Table 2. The results of the harmonic analysis of the monthly mean sea level. Here, $h=h_0+h_1\cos(\sigma t-\mu_2)+h_2\cos(2\sigma t-\mu_1)$, $\sigma=2\pi/12$ months, and the units are cm and degree.

	1951		1952		1953		1954		1955		1956	
	h_0		h_0		h_0		h_0		h_0		h_0	
Owase	—		—		—		− 5.8		− 8.8		−15.0	
Uragami	−15.7		−18.8		−15.2		−13.8		−17.9		−23.2	
Kushimoto	− 8.4		− 7.1		− 7.3		−10.2		−15.1		−10.7	
Shimotsu	7.1		—		—		9.9		—		6.9	
	h_1	μ_1	h_1	μ_1	h_1	μ_1	h_1	μ_1	h_1	μ_1	h_1	μ_1
Owase	—	—	—	—	—	—	12.8	265–30	5.5	243–00	11.6	249–10
Uragami	10.1	256–10	9.6	259–50	17.9	253–30	11.3	271–00	3.6	266–50	11.6	252–30
Kushimoto	12.5	205–10	15.4	237–30	11.9	242–20	11.7	270–00	2.6	276–40	13.9	233–50
Shimotsu	14.0	207–40	—	—	—	—	12.6	260–50	—	—	12.3	232–20
	h_2	μ_2	h_2	μ_2	h_2	μ_2	h_2	μ_2	h_2	μ_2	h_2	μ_2
Owase	—	—	—	—	—	—	1.4	294–50	2.8	122–00	4.4	232–20
Uragami	3.6	159–10	3.4	178–20	3.1	121–40	2.2	201–50	1.2	131–40	4.7	231–50
Kushimoto	3.7	120–40	2.4	152–20	2.0	159–50	1.7	234–30	0.9	102–30	5.9	229–50
Shimotsu	2.9	133–40	—	—	—	—	1.3	296–30	—	—	4.7	207–40

tion of the monthly mean sea level and the amplitude of which is considerably large.

The amplitude of the annual variation is much larger than that of the semi-annual one in general. This is the same relation as is found between S_a and S_{sa}. But, an exception is seen in 1955, when the annual one was very slight as compared with that in other years and the amplitude of the semi-annual one was from a third to a half of that of the annual one. In the year, the cold water region was large in winter, while it was very small from summer to autumn. And so, the sea level in winter was not so low as that in other years under the influence of the striking westward current of low density water along the east coast of the Kii Peninsula. On other hand, as the cold water region was very small in summer and the current axis was situated at about 33°N from summer to autumn, the sea level did not become higher in autumn when the maximum value generally occurred. Therefore, the abnormal variation in the year may have a close relation to the thermal effect and the fluctuation of the oceanographic conditions. Though this is not so remarkable in other years as in the year, the relation is also found in other years.

Since the annual variation of the monthly mean sea level varies from year to year, S_a and S_{sa} are also expected to be variable. It was already shown by MIYAZAKI (loc. cit.) that S_a tide at Kushimoto fluctuated remarkably from 1925 to 1936. Accordingly, the departure from the predicted sea level will depend upon the change of S_a tide, too. S_{sa} tide is generally small and the effect of its variation is not significant.

7. Concluding remarks

The fluctuation of the monthly mean sea level at four tidal stations around the Kii Peninsula is discussed, connected with the oceanographic conditions. Pressure effect is eliminated by using the value reduced to the standard atmospheric pressure. The comparison of the dynamic height anomaly at G_1 with the monthly mean sea level at Kushimoto shows that the sea level at the tidal stations in this region corresponds well to the oceanographic conditions. In this case, the influence of the density change is not eliminated, but this good correspondence means the equality of the effect. However, the annual

variation of the density due to the thermal effect etc. is not constant for each station. In order to eliminate it, we use the departure of the monthly mean sea level reduced to the standard atmospheric pressure. Thus, when the departure is discussed, the influence of the oceanographic conditions is most predominant in the various effects. This influence, however, means the variation of the density. For instance, the northward movement of the current axis causes the decrease of the density and so makes the sea level rise. Consequently, the influence may be regarded as one owing to density change except for owing to the thermal effect or the inflow of the land water which is considered to show the annual variation. Since the fluctuation of the monthly mean sea level is discussed, the fluctuation of small dimension is, of course, escaped from this consideration. Though Windstau is very important for the sea level, the influence upon the monthly mean sea level seems not to be significant. This is ascertained from the result that the fluctuation of the monthly mean sea level is nearly accounted for by the atmospheric effects, the density change and the oceanographic conditions even when the Windstau is not taken into account.

The difference in the monthly mean sea level between Kushimoto and Uragami is also in close relation to the oceanographic conditions, and sometimes amounts to an astonishing value in spite of the short distance between them. Though the reason cannot be made clear, we shall discuss it in another paper.

Furthermore, the harmonic constituents of long period, solar annual tide (S_a) and solar semi-annual tide (S_{sa}), are expected to fluctuate pretty from year to year under the influence of the fluctuation in the oceanographic conditions.

Acknowledgements: The present author expresses his hearty thanks to Dr. K. HIDAKA, Dr. K. YOSHIDA and Dr. T. NAN'NITI for their kind advice and guidance, and to Dr. Y. SHIBATA and Dr. K. HISHIDA for their encouragement. And he is indebted to Mr. H. YOSHIZAWA for furnishing materials and to Mr. Y. SHIMOMURA for preparing this manuscripts.

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Drop Production by Bursting of Air Bubbles on the Sea Surface (1) Experiments at Still Sea Water Surface*

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Abstract: Aspects of the drop production at a still surface of sea water by bursting of single air bubbles smaller than several mm in diameter are observed. The height of drop ejection, size of the drops and the efficiency of drop ejection are shown as a function of bubble size, for different temperatures. It is found that a remarkable critical diameter of a bubble exists with regard to the feature of the drop production. This critical diameter is about 4 mm.

Some characteristic features in the case of flocking bubbles on the sea water surface are also observed.

1. Introduction

Sea water droplets ejected from the sea surface, after evaporated individually or modified by separation, supply a significant portion of the air borne condensation nuclei which dissolve into raindrops and fall to the ground.

It was suggested early that such sea water droplets are produced by bursting of the air bubbles formed by trapping of air in breaking waves. A. H. WOODCOCK and his collaborators (A.H. WOODCOCK *et al* 1953; C. F. KIENTZLER *et al* 1954) performed high speed photographic studies on the bursting of air bubbles, and confirmed that disintegration of the vertical water jet which forms upon collapse of the bubble cavity, is effective mechanisms for production of such sea water droplets. B.J. MASON (1954), D.J. MOORE and B.J. MASON (1954) and F.N. KNELMAN *et al* (1954) showed, besides the jet mechanism, that a large number of small droplets are formed by shuttering of the bubble film. Recently, D.C. BLANCHARD and A.H. WOODCOCK (1957) studied four natural mechanisms for production of these bubbles, namely, whitecaps, rain, snow and supersaturation of the surface water of the sea.

They believe that sea water droplets containing the sea salt of from 10^{-8}gm to 2×10^{-14}gm or smaller, are produced by disruption of the liquid jet or disintegration of the bubble film. H. DESSENS (1946; 1949), however, described on the possibility that one droplet is reduced to much smaller nuclei by splitting when it crystallizes suddenly while it flies during many days accompanied by crystallization and dissolution over and over again. S. TWOMEY and K.N. MCMASTER (1955) showed that one droplet produced many hundreds of very small nuclei when it crystallized.

It was found, moreover, by geochemists by chemical analysis of the atmospheric particles and precipitation, that the contents of the air borne particles vary according to the distance from the sea, to the height, or to the district. So that it well be considered that sea water droplets ejected from the sea develop into various atmospheric particles through complicated processes which still remain unknown.

The laborious works of WOODCOCK and his collaborators are mainly restricted to the bubbles of smaller diameters, but droplets of few tenth millimetre in diameter may also be carried by an atmospheric turbulence or by a large-scale ascending current for a great distance, and, during their stay in the air, evaporate and reach high altitudes. Evaporation of these droplets means water transport in the form of larger mass from the ocean to the atmosphere on the one hand, and presumably the great source of the air borne salt on the other.

Under these considerations the authors began to study the drop production by bursting of air bubbles including those of larger diameters.

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2. Apparatus and Experimental Procedure

The experiment consists of two parts. The one is concerning to the bubbles that burst at the sea water surface individually, the other is to study the effect of flocking of the bubbles.

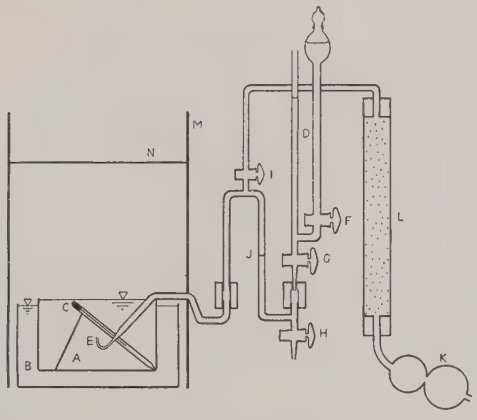

Fig. 1. Layout of the apparatus for the study of the bursting of separate bubbles.

Fig. 1 illustrates schematically the layout of the apparatus to study the bursting of individual bubbles. The glass tank A contains sea water, the other water tank B holds ice and water or warm water, so as to keep the sea water in A at constant temperature. Experiment was performed at three different temperatures, namely, 4°C, 16°C and 30°C approximately. The salinity of the sea water used was 35.1 per mille. The part D is a microburette, E is a glass jet. If one shut the cocks F, H and I, and drop the water through the cock G slowly, the water head in J arises and the same volume of air as the dropped water leaves from the jet E in the form of bubbles. The air that forms the bubbles was supplemented from the spray K by the cock I through the tube L which contains silica-gel; the water in the tube J was drained through the cock H. Silica-gel was used to prevent the jet E from clogging due to condensation of the water vapor in the air when the temperature of the sea water was low.

Several jets of different bores were now used which produced bubbles from 1.1 to 6.0 mm in equivalent diameter for spherical bubbles. When one of the jets was set and the water in the microburette D was dropped at a constant rate, bubbles of a constant diameter leaved from the jet E separately with a constant time interval, so that we could calculate the bubble volume, consequently, the bubble diameter by counting the bubble number and the water volume that dropped from D.

M is a frame with four pillars; a filter paper specially treated with a dye (cf. Y. Toba 1958), and a glass plate N was mounted at different heights on this frame and received the drops ejected from bursting bubbles, thus the height of ejection and the size of the drops, and the efficiency of drop ejection were measured. To read the stains on the dyed filter paper, a reading microscope with a 1/20 mm vernier was used.

For an experiment on the flocking bubbles, the air of constant pressure, consequently, of constant flux which came through pressure regulators from an air container, was let in through a hole 4 cm below the sea water surface. Eight sorts of bubbles from 2.1 to 7.4 mm in diameter were ejected respectively at a rate from 13 to 45 per sec, the filter paper was set at various heights and received the drops for some measured time intervals. The stains on the filter paper, in this case, were read by a scale which enables us to read them in the drop diameter to 0.1 mm, and counted for each drop diameter grade. Bubble diameter, in this case, were measured by a photographic method, and converted into the equivalent diameter for spherical bubbles.

3. Results and discussion

Fig. 2, Fig. 3 and Fig. 4 illustrate the height of ejection of the drops, the size of them and the efficiency of the drop ejection, respectively, by individual bubbles bursting at the sea or fresh water surface as a function of bubble diameter, for the water temperature of 4°C, 16°C and 30°C. For the sake of comparison, the results of Blanchard and Woodcock (1957) are shown together in Figs. 2 and 3.

In the region where the both data overlap, they agree, although the methods of measurement in both cases are different. In the neigh-

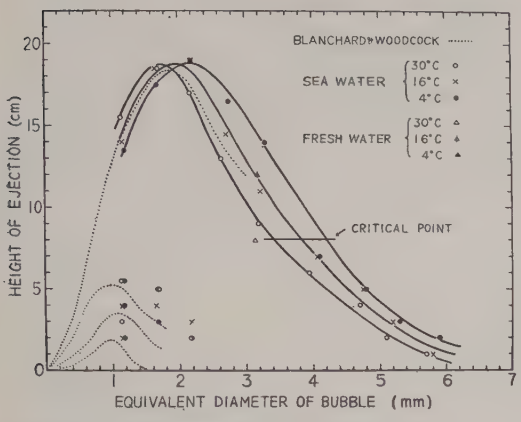
BLANCHARD & WOODCOCK .....
SEA WATER 30°C 16°C 4°C
FRESH WATER 30°C 16°C 4°C
CRITICAL POINT
HEIGHT OF EJECTION (cm)
EQUIVALENT DIAMETER OF BUBBLE (mm)

Fig. 2. The height of ejection of the drops from bubbles of a determinate size bursting at the surface of sea water, and of fresh water as a function of bubble diameter. BLANCHARD-WOODCOCK's data (1957) are entered as dotted lines.

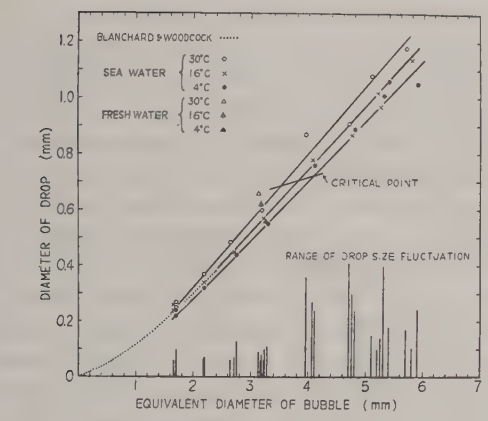
BLANCHARD & WOODCOCK .....
SEA WATER 30°C 16°C 4°C
FRESH WATER 30°C 16°C 4°C
CRITICAL POINT
RANGE OF DROP SIZE FLUCTUATION
DIAMETER OF DROP (mm)
EQUIVALENT DIAMETER OF BUBBLE (mm)

Fig. 3. The diameter of drops that are ejected from bubbles of a determinate size bursting at the surface of sea water, and of fresh water as a function of bubble diameter. For bubble diameter of 1.7 mm, marks of second drops are also entered. Sticks standing on the abscissa of this graph indicate the range of drop size fluctuation for each mark which represents the mean value. BLANCHARD-WOODCOCK's data for top drops (1957) are entered dotted line. (The mass of salt in their data are converted in this figure into drop diameter by estimating the salinity to be 33 per mille.)

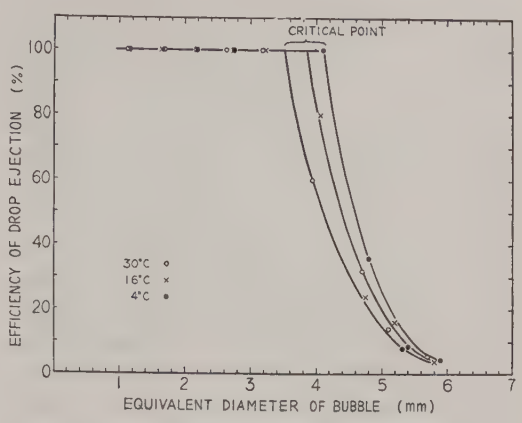

Fig. 4. The efficiency of drop ejection by bubbles bursting at the surface of the sea water as a function of bubble size.

bourhood of the bubble of approximately 1.7 mm in diameter, two points are entered in Fig. 3 for each temperature, the upper representing the second drops. This is also close to the data of BLANCHARD and WOODCOCK (1957).

Each bubble less than about 2 mm ejected four or less drops, but the ejection of the second to the fourth drops, in our observed range, showed less regularity. Bubbles of smaller diameters than about 4 mm ejected top drops without fail, and the height of ejection of these drops was more regular, except the case described later. For the diameters larger than about 4 mm the efficiency of drop ejection falls rapidly from 100 per cent to several per cent in the case of the bubbles of 6 mm in diameter as shown in Fig. 4. In this case, only several bubbles among one hundred bubbles eject drops, in other words, the majority of the bubbles burst in vain with respect to the drop production.

This critical diameter appears clearly in Figs. 2 and 3. As the bubbles larger than this diameter eject drops to random heights, the curves of Fig. 2 on the right of that point were drawn to show the maximum height observed.

Each point in Fig. 3 represents the mean of the observed values. With regard to the bubbles of smaller diameter than the critical one, each marks lie closely on the lines in the

figure, and the range of the drop size fluctuation is also small as shown, but on the larger side of the critical point, the range of the drop size fluctuation suddenly becomes large and the mean values scatter greatly.

O. Stuhlman Jr (1932) stated on the drop ejection from bubbles in distilled water at 21°C, that there was a maximum of the height of ejection at bubble diameter of 1.2mm, and larger bubbles than this size burst with less regularity. He attributed this irregularity to the instability of the rising bubble, but did not describe more about the critical point as we did above.

It may be inferred that the form of a floating bubble is determined by the equilibrium condition of the buoyancy of a bubble and the surface tension; that is, a smaller bubble has the type like the left of Fig. 5, whereas

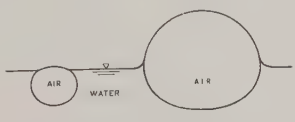

Fig. 5. Two types of the form of a floating bubble.

a larger one has that like the right. The bubble form that lies between these two types has a diameter of about 4 mm, and probably forms a critical state in the mode of bubble bursting. This point will be discussed theoretically in a succeeding paper by the junior author. As surface tension decreases with increasing temperature, bubbles with constant volume rise more above the water surface when the temperature is higher, so that the bubble diameter that lies between the two types will become smaller for higher temperature, this agrees with the fact that the critical diameter moves left in figures with increasing temperature. The potential energy of a drop calculated from the height of ejection is shown in Fig. 6; the energy is nearly a function of bubble size only and not of temperature, as may be expected. The amount of the surface energy of the submerged portion of a bubble was also calculated, and is shown in dashed lines in Fig. 6. It will be seen that only few per cent of the energy of a bubble is spent for drop production. Details of this problem will be discussed in a succeeding paper.

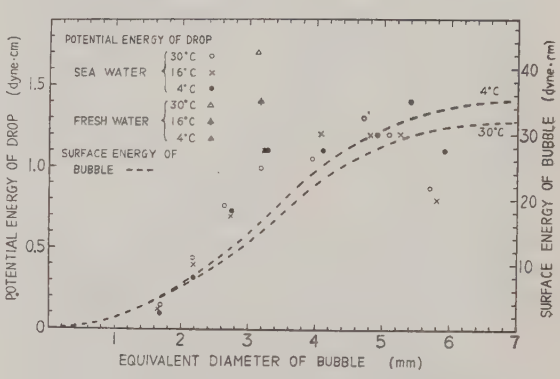

Fig. 6. Comparison of the potential energy of a drop ejected from a bubble with the surface energy of the submerged portion of the same bubble. As to the bubbles having larger diameters than the critical point, the energy of a drop is computed for one which reached the maximum height.

The process of bubble bursting is very complicated and will be affected by many disturbing factors. Nevertheless, the observed regularity appeared in the mean values indicates that some definite physical process underlies. In this respect, some peculiar phenomena observed in our experiment will be mentioned, though their reason is unknown: many of the bubbles of 3.4 mm in diameter, which is somewhat smaller than the critical diameter, at 4°C, burst as soon as they reached the sea water surface, did not remain floating stably, and their drop ejection was irregular; bubbles of 2.2 mm ejected at 30°C, the extremely small second drops to the height of 2 cm regularly, and even larger drops than the top drops to the height less than 1 cm.

Experiments were also made in pure water for such bubble diameter as shown in Fig. 2 and Fig. 3 at three different temperatures. In this case, the bubbles burst as soon as they reached the water surface at 30°C with few exceptions, and they ejected drops to more irregular heights; many of the bubbles burst as soon as they reached the water surface at 16°C, and every bubble burst after floating for a long while (10 to 40 sec) at 4°C.

When many bubbles exist at the same time on the water surface, some of the bubbles flock and some unite in a single bubble. We shall report some of the results obtained on the bursting of flocking bubbles in the following. The experiment was made at about 10°C.

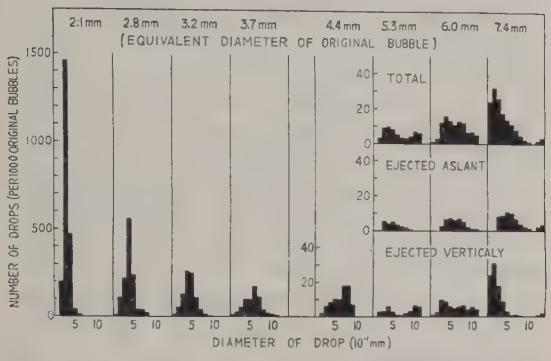
2.1mm 2.8mm 3.2mm 3.7mm 4.4mm 5.3mm 6.0mm 7.4mm
(EQUIVALENT DIAMETER OF ORIGINAL BUBBLE)
TOTAL
EJECTED ASLANT
EJECTED VERTICALY
NUMBER OF DROPS (PER 1000 ORIGINAL BUBBLES)
DIAMETER OF DROP (10⁻¹mm)

Fig. 7. The size distribution of the drops ejected from the flocking bubbles on the sea water surface for each of the eight diameter grades.

Fig. 7 illustrates the size distribution of the drops ejected from the flocking bubbles for each of the eight diameter grades. From this figure one may find (1) that the total number of the drops decreases with an increasing bubble diameter; (2) that drop size becomes scattered over wide range with an increasing bubble diameter; (3) that the drops ejected aslant appear when the bubble diameter is greater than about 4 mm. Such a drop may be considered to be produced when a bubble burst as it is entrapped into the cavity of the adjacent bubble which burst immediately before; (4) that the total number of drops falls off sharply when the bubble diameter becomes greater than about 4 mm; (5) that the second or the third maxima appear on the drop size distribution for bubbles of 3.7 mm or more in diameter; and (6) that the number of the small drops increases again for a large bubble diameter, so that it is considered that there remain small bubbles after large bubbles burst. From (1) and (2) we may say that the unification of bubbles increases with an increasing diameter, so that the effective number of bursting bubbles decreases. (3) and (4) indicate the efficiency of drop ejection and the existence of the critical diameter already mentioned. From (3) and (5) it follows that the aspects of bursting and of drop production of flocking bubbles differ, in part, from those of individual bubbles. With respect to (6), we observed actually that over ten small drops, less than 2 mm in diameters, remained after a large bubble of 1 cm or more burst.

The drop sizes corresponding to the maxima of the first series in drop size distribution in Fig. 7 agree with those in Fig. 3.

Fig. 8 illustrates the number of drops that are ejected from flocking bubbles of 2.8 mm in original diameter and are caught by the filter paper at the various heights, for each grade of the drop diameter. This shows that, in the case of flocking bubbles, the height of ejection of drops for each grade of drop diameter is distributed nearly uniformly from the water surface to a certain height. Fig. 7 is based on a series of these figures. The

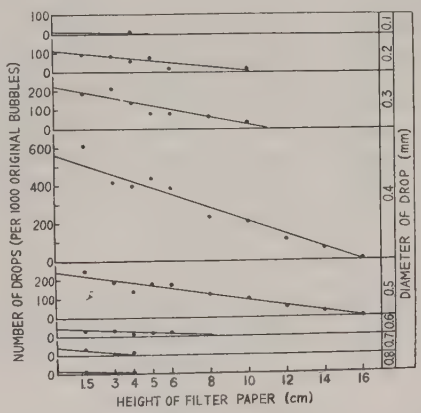

Fig. 8. The numbers of drops that are ejected from flocking bubbles of 2.8 mm in original diameter and are caught by the filter paper at the various heights, are illustrated for each grade of drop diameter.

relation between the maximum height of ejection and drop size also somewhat differs from that obtained by combination of Fig. 2 with Fig. 3, and the main feature of this difference is attributed to the drops ejected aslant. From this relation and from Fig. 7 it may be seen that drops of 0.2 to 0.3 mm

in diameter are the most prominent in the production of droplets by the bursting of air bubbles.

The suspension of discrete particles in fluid is governed by their terminal velocities and turbulence in the fluid. It is interesting to note that the terminal velocities of water drops of 0.2 to 0.3 mm in diameter are equal to those of sand grains of about 0.1 mm in diameter which are the transitional grains between dune sand and loess and are easily picked up by wind to an appreciable height.

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Observation of Sea Water Droplets by Filter Paper*

Yoshiaki TOBA**

Abstract: A general notice on the method to measure the size of water drops by filter paper is first described. Then the process of preparation of a dye-treated filter paper, which is suitable for the observation of sea water droplets, is reported.

1. Methods of Observation of the Size of Water Drops

For estimation of the size of water drops, several methods have been so far employed, including the flour-method to measure a dough-pellet which is formed by a water drop when it fell into fine, uncompacted flour (W.A. BENTLEY 1904), the use of a sooted screen by measuring the round mark formed on it after a water drop passed it (D.C. BLANCHARD 1946 b), photographic measurements, and the use of filter paper.

It is said that the method to estimate the drop size from the area which is occupied by absorbed water of a drop on filter paper, was introduced by J. WIESNER (1895). Later, some sort of dyes came to be used for preserving the stains on the filter paper. Some sprinkled dye powder over the filter paper, others dyed the water itself previously before experiments were carried out, while the other treated the filter paper with dyes so that it was dyed at the very spot where a water drop fell. As to dyes, eosine (P. LENALD 1904; A. DEFANT 1905), water soluble, blue, anilin dye (R.H. MAGARVEY 1957), potassium permanganate (L.J. ANDERSON 1948), gentian-violet dye (J.S. MARSHALL *et al* 1947), methylene blue dye (D.C. BLANCHARD 1953) and others were used for fresh water drops.

To measure sea water drops, BLANCHARD and WOODCOCK (1957) used methylen blue dye by dying the sea water itself. S.G. BOYCE (1951) used a filter paper which was dipped in 0.01 N K_2CrO_4, then dried in 0.02 N $AgNO_3$, subsequently put in distilled water to remove the exess $AgNO_3$ and redried. This filter paper leaves light-yellow spots by sea water droplets.

The filter paper method has been used by many investigators, especially, for the observation of rain drops. It has been confirmed since WIESNER that a filter paper must be of the same standard, that it must be as homogeneous as possible and that it should be dried sufficiently before it receives water drops, and it has been further noticed that the settling velocity of a drop affects the spot size for drops lager than about 0.7 mg in weight, or about 1.3 mm in diameter (L.J. ANDERSON 1948; R.T. JARMAN 1956; R.H. MAGARVEY 1957).

Now, let M denote the drop volume in mm^3, A: the area of a spot in mm^2, and α and ν: the thickness in mm and the porosity of the filter paper, then it is

$$M=\alpha\nu A \qquad (1).$$

As spots left on filter paper generally have an elliptic form, it follows that

$$M=C_1D^a \qquad (2),$$

or

$$d=C_2D^b \qquad (3),$$

where D is the longer diameter of a spot in mm, d is the diameter of a drop assumed as sphere in mm, and C_1, C_2, a and b are numerical constants. It is theoretically inferred that a equals 2 and b equals 2/3. By direct measurement of d and D, one may obtain C_1 and C_2 for his filter paper.

Recently, R.T. JARMAN (1956) examined the spreading area of water on a filter paper and obtained the value of $b=0.84\pm0.02$, and pointed out that the value of b is not in all cases 2/3. R.H. MAGARVEY (1957) obtained, for his case, the following relation for water drops

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between 0.5 mm and 10.5 mm in diameter,

$$d=\frac{1}{3}D^{0.75} \quad (d>1.5),$$

$$d=\frac{1}{3}D^{0.93} \quad (d<1.5).$$

Thus, it should be kept in mind that the value of b may be varied according to the kind of liquid forming a drop, the nature of filter paper used, the relation between drop size and paper thickness, and especially for large drops, condition of drying during absorbtion of water, or the settling velocity of a drop, and so forth.

In our country, H. MARUYAMA and K. HAMA (1954) proposed a method of observing precipitation such as rain, snow etc. by using water blue and Toyo Filter Paper No. 1511. The last two figures of the number specifying this paper represent the thickness of the paper. So, in this case, it has the thickness of 0.11 mm. Water blue was milled to fine powder and was suspended in benzine petroleum. The filter paper was passed through the suspension, and dried. Thus the dye particles were scattered uniformly in the fibrous tissue and fixed there. Water drops that fell upon it, formed blue spots of elliptic shape on the paper. They obtained from direct observations, the relationship,

$$\alpha v=0.0343$$

and from measurement of the drops of $d=0.4$ to 6 ($C=2$ to 70),

$$M=0.0230\,D^2 \qquad (4)$$

and

$$d=0.353\,D^{\frac{2}{3}} \qquad (5).$$

On this filter paper, however, only faint marks are formed by droplets of sea water. Probably, this is due to the fact that water blue dissolves very little in sea water owing to salting out. Attempts to vanish salting out by using some surface active agents, and tests on various dyes were carried out in our laboratory, but the dyes suitable to the sea water were not many. Only the following treatment of filter paper seems to be suitable for sea water droplets.

2. The Process of Preparation and the Use of Dye-Treated Filter Paper

About 2 gm of Nippon Sky Blue 390% (direct sky blue 5 B; manufactured by Sumitomo Chemical Co., LTD.) are crushed in a mortar under the presence of such a quantity of ethanol as immerse the dye powder. In the course, ethanol is added three or four times during thirty or forty minutes. Then, it is poured into a small beaker by rinsing the mortar with exess ethanol of about 20 cc. In tens of minutes, the dye particles precipitate considerably, and dark red fluid is left. After leaving it one or two days, the superficial dark red fluid is removed as completely as possible, softly with an injector, and the sediment is poured into a large beaker containing benzine ptroleum of about 400 cc in which surface active agent of about one per cent in weight of benzine petroleum is dissolved, and stirred. By this process, dye particles suspend fairly well in benzine petroleum. After leaving for a few tens of minutes, it is poured into an enameled dish. Toyo Filter Paper No. 1511 (20 cm×20 cm) is passed through the suspension quickly piece by piece, and dried. As the benzine petroleum in the dish evaporates, it has to be replenished in due quantity. While 300 cc of the liquid is supplied, 50 pieces of the dye-treated filter paper are prepared.

For a surface active agent which keeps the dye particles in suspension, powdered matter is desired, and Neogen Powder (40% of Na-alkylbenzene-sulfonate is contained; Dai-ichi Kogyo Seiyaku Co., LTD., Kyoto) gave the finest result in our experiment.

The prepared filter paper leaves distinct marks by sea water droplets. Moreover, we can easily distinguish the stains by sea water droplets from those by fresh water droplets: the former are uniformly colored, while the latter are enclosed by links of dark color near the borders (Fig. 1).

If we determine the coefficients in equation (2) and (3) in the manner described in the next section, we can determine the size of the original sea water droplets, by means of the longer diameters of the stains on the paper.

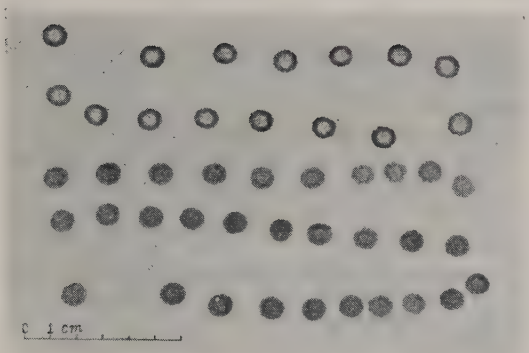

Fig. 1. The difference between the marks by sea water drops and those by fresh water drops. The upper two ranks are the stains by fresh water droplets, and the lower three ranks are those by sea water droplets.

The diameters of the stains can easily be measured by a reading microscope with 1/20 mm vernier. When the size distribution of a number of drops is desired, it may be conveniently obtained by the scale which is marked with the values of D corresponding to d, for example, of 0.15 mm, 0.25 mm, 0.35 mm, ..., successively.

Character of this paper does not sensibly change for ordinary preservation, but it is desired to be preserved in dry places.

The marks by sea water droplets on this paper are very distinct, and it is readily used for ordinary sea water droplets. Examples of the application of the paper will be published in a succeeding report concerning the drop production by the bursting of air bubbles on the sea surface.

3 The Determination of the Coefficients and the Range of Application of the Paper

The constants in equation (2) and (3) may vary according to the kind and quantity of the dye or of the surface active agent used, or to the salinity of the drops. It is desirable, therefore, that the constants are determined by the user for his own case. Since MARUYAMA and HAMA showed already that the relationships (4) and (5) hold over a fairly wide range of M, it may be sufficient to determine the coefficients C_1 and C_2 by the values of M for some range of drop diameter, assuming a and b to be 2 and 2/3, respectively.

By using a microburette, M or d is determined, and by actually measuring the corresponding D, we may obtain C_1 and C_2 from the relations,

$$C_1 = M/D^2 \qquad (6),$$

and

$$C_2 = (6C_1/\pi)^{\frac{1}{3}} \qquad (7).$$

The author determined the coefficients for the paper above described for sea water droplets, and the values $C_1=0.027$, $C_2=0.37$ were obtained. For fresh water drops, $C_1=0.034$ and $C_2=0.40$ were also obtained.

The droplets larger than 0.2 mm in diameter can be measured sufficiently by the paper. To estimate the effect of salinity, stains were made by droplets of various salinity. Fig. 2 illustrates them, showing that the filter paper can well be used within the salinity range of from 0 to 100 per mille.

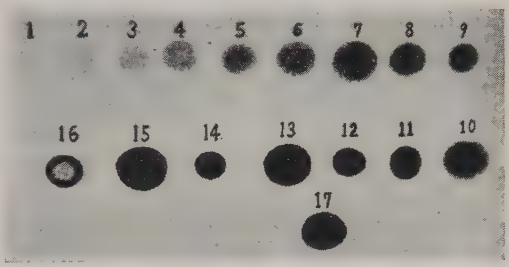

Fig. 2. The variation of the nature of the stains with salinity. The salinity of droplets are as follows:—Nos. 1 (200), 2 (180), 3 (160), 4 (140), 5 (120), 6 (100), 7 (90), 8 (80), 9 (70), 10 (60), 11 (50), 12 (40), 13 (30), 14 (20), 15 (10), 16 (0), 17 (34.7: sea water). (The figures in parentheses are salinity in per mille.)

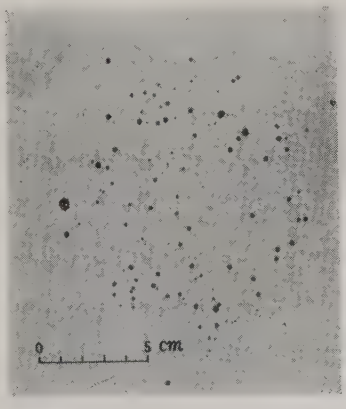

Fig. 3. Examples of application to the observation of sea water droplets produced by bursting of air bubbles on the sea water surface.

Acknowledgements

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本邦内湾の海洋化学的研究 (第3報), 塩類組成の変動と海況との関係, (その 1, 浦戸湾について)*

山　本　広　志**

Oceanographical Studies on Japanese Inlets (3), Relations between Variation of Chemical Compositions and Hydrographical Conditions in Urado Bay (Part 1)

Hiroshi YAMAMOTO

Abstract: In order to study the intrusion of sea water into inlets, chemical elements which usually found in sea water are measured in some inlets of Japan, with emphasis on the ratio of chlorinity between the sea and inlet waters. And to show how the value of the ratio varies according to the kinds of chemical elements, hydrographic conditions, and various characteristics of paricular inlets, the author made researches on calcium and magnesium in Urado Bay. The findings are as follows:—

1. Mixture of calcium shows a tendency to lag behind that of chlorinity in its earlier stages, while in its later stages it becomes quicker.
2. The quantity of calcium varies according to hydrographic conditions, even though the quantity of chlorinity to be the same.
3. The ratio of chlorinity to magnesium stays almost uniform irrespective of the variation in chlorinity. There cannot be found any appreciable difference due to hydrographic conditions.

1. 緒　言

海洋に於ては海水の主要塩類はその組成が一様であることが知られているため, これらの化学元素と塩素量との比をもつて塩類組成を表はしている. 従つてこの塩素比の変動は異水系の消長及びそれらの境界の変化などを考える上に重要であり, この観点から種々な報告がなされている.

然しながら, この様な水塊分析を行うためには主要塩類を含めてそれ以外の化学元素相互の関係やそれらと塩素量との関係についても検討する必要がある.

著者等は環境を異にする内湾で化学元素の塩素比を測定して, 化学元素の種類による塩素比の変動の差異, 及びそれらが内湾の特質と如何なる関係を有するかに注目して研究を行つてきた. 今回は浦戸湾に於てこれらの化学元素の内カルシウム及びマグネシウムの塩素比を測定しその変動について考察したので報告する.

2. 研究方法

前報に述べた如く, 浦戸湾には多くの陸水が流入しているので潮汐の影響により内湾水の濃度も異るため, 鏡川水系及び国分川水系より湾口にかけて種々な地点に於て水深 1 mおきに採水を行い分析した. 又張潮時, 落潮等の海況の異なる場合についても検討した. 分析の方法は EDTA 法によつた, 次にその概要を示す.

〔Ca+Mg〕; 海水の一定量につき, 塩化アンモニウム及びアンモニウムの緩衝液で pH を 10 に調節し EBT を指示薬として EDTA で滴定し合量を求めた.

〔Ca〕; 海水の一定量に溶存するカルシウムに当量よりやや過剰の EDTA をあらかじめ添加し, しかる後苛性ソーダを用いて pH を 12 に調節して生ずる $Mg(OH)_2$ の沈降をまつて, これに濃度既知のカルシウム溶液の一定量を添加し指示薬としてドータイト NN (同仁薬化製) を加えて滴定を

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行い，滴定結果からあらかじめ添加したカルシウム量を差引くことにより求めた．濃度既知のカルシウムを添加することはあらかじめ添加するEDTAによつて一部生じるマグネシウムのキレートを $Mg(OH)_2$ として完全に分離せしめるためである．

〔Mg〕；前記合量からカルシウム量を差引く事により求めた．

比較のためにこのような方法で測定した太平洋の資料について分析した結果を第1表に示す．

Table 1.

Sample No.	Date	Location	Temp.	Cl g/l	Ca mg-atom/L	Mg mg-atom/L	Ca/Cl	Mg/Cl
1	11.15	31°-08′N 135°-28′E	23.7	19.36	10.29	52.28	0.02129	0.06605
2	11.15	29°-13′N 135°-00′E	23.6	19.30	10.27	52.60	0.02134	0.06628
3	11.16	27°-00′N 135°-00′E	24.2	19.32	10.14	52.08	0.02122	0.06557
4	11.17	25°-00′N 135°-04′E	25.2	19.42	10.35	52.57	0.02134	0.06584
5	11.23	19°-08′N 131°-41′E	27.7	19.35	10.25	52.48	0.02117	0.06594
6	11.25	21°-28.5′N 133°-13′E	27.0	19.50	10.33	52.69	0.02124	0.06571
7	11.26	20°-43′N 133°-20′E	25.8	19.43	10.29	52.47	0.02122	0.06569
8	11.27	22°-31′N 133°-10′E	26.4	19.48	10.33	53.13	0.02125	0.06633
9	12. 6	21°-34.5′N 132°-47′E	26.7	19.43	10.26	52.86	0.02116	0.06615
10	12. 9	21°-52′N 132°-58′E	26.4	19.46	10.33	52.14	0.02121	0.06516

3. カルシウムの塩素比について

浦戸湾での測定結果を第2表及び3表に挙げた．海水中の化学組成が変動してゆく過程を考察するために湾全域の資料について塩素量に対応する化学元素の塩素比をプロットすると混合に際しての化学元素の増減がみられて便利である．このためカルシウムについて示すと第1図の如くなる．

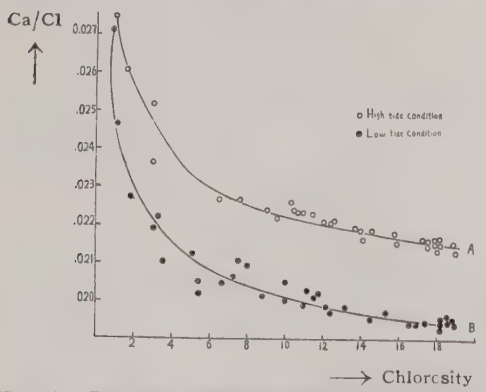

Fig. 1. Relation between Chlorosity and Ca/Cl.

図からも明らかな如く塩素度として 2～7 までは急降し，それ以上は徐々に下降して一定値に近ずく．即ちこのことは，陸水が湾内水に流入して次第に混合される場合，塩素の混合の度合に比してカルシウムの混合の度合が異なり，安定な内湾

Table 2. (May 5, 1957)

St.	Depth. m	Temp. C°	Cl. g/l	Ca mg-atom/L	Mg mg-atom/L	Ca/Cl	Mg/Cl
〔1〕	0	16.7	3.56	1.87	9.97	0.02105	0.06811
	1.2	19.9	17.08	8.30	46.82	.01948	.06667
〔2〕	0	17.6	6.84	3.60	18.81	.02109	.06688
	0.9	19.4	14.61	7.11	40.18	.01951	.06688
〔3〕	0	16.9	5.51	2.82	15.26	.02051	.06735
	3.8	19.4	17.03	8.25	46.79	.01942	.06682
〔4〕	0	17.6	6.85	3.50	18.92	.02048	.06717
	1	17.7	8.99	4.51	24.78	.02011	.06704
	2.9	19.3	17.23	8.39	47.51	.01952	.06706
〔5〕	0	18.1	7.37	3.79	20.29	.02061	.06695
	1	17.7	11.70	5.88	32.16	.02014	.06685
	2	18.32	12.63	6.29	34.68	.01994	.06678
〔6〕	0	18.4	8.13	4.25	22.25	.02095	.06656
	1	18.3	12.32	6.61	33.70	.01988	.06652
	2	19.0	16.66	8.16	45.56	.01963	.06651
	3	19.2	18.17	8.89	49.70	.01961	.06652
	4	19.1	18.75	9.14	51.39	.01951	.06666
	5	19.1	18.73	9.00	51.28	.01926	.06658
	5.5	19.3	18.75	9.09	51.54	.01943	.06685
〔7〕	0	18.4	8.95	4.48	24.73	.02006	.06720
	1	17.8	11.80	5.85	32.42	.01987	.06684
	2	18.9	15.59	7.68	42.54	.01974	.06636
	3	19.1	17.24	8.36	47.27	.01944	.06668

Table 2. (*Continued*)

St.	Depth. m	Temp. C°	Cl. g/l	Ca mg-atom/L	Mg mg-atom/L	Ca/Cl	Mg/Cl
	4	19.1	18.65	9.07	51.19	0.01949	0.06675
	5	19.2	18.62	9.00	51.12	.01935	.76677
	6	19.0	18.79	9.19	51.57	.01943	.06675
〔8〕	0	18.4	10.90	5.49	29.91	.02019	.06674
	1	18.1	12.50	6.15	34.30	.01972	.06673
	2	18.4	14.64	7.20	40.11	.01971	.06663
	3	18.8	18.13	8.84	39.58	.01954	.06651
	4	18.8	18.72	9.05	51.60	.01938	.06703
	5	18.8	18.83	9.07	51.84	.01931	.06695
	6	18.8	18.89	9.12	52.02	.01935	.06697
	6.5	18.6	18.94	9.14	52.20	.01934	.06703
〔9〕	0	18.9	11.59	5.74	31.94	.01985	.06702
	1	19.0	12.20	6.01	33.63	.01974	.06704
	3	18.7	18.61	9.05	51.18	.01949	.06628
	4	18.7	18.83	9.15	51.67	.01948	.06673
	5	18.6	18.81	9.13	51.78	.01945	.06695
	6	18.5	18.93	9.18	52.03	.05944	.06682
	7.3	18.5	18.93	9.18	52.03	.01944	.06682
〔10〕	0	17.9	12.66	5.76	32.12	.01980	.06699
	1	17.8	12.67	6.23	34.72	.00971	.06665
	2	17.9	16.42	8.08	45.14	—	—
	3	17.9	18.45	8.95	50.68	.01944	.06680
	4	17.9	18.66	9.17	51.11	—	—
	5	18.2	18.81	9.18	51.73	.01956	.06688
	6	18.0	18.82	9.21	51.73	.01961	.06685
	7	18.1	18.91	9.27	51.4	.01965	.06680
	9	17.9	19.02	9.31	52.14	.01962	.06667
	9	17.9	19.02	9.30	52.33	.01960	.06691
	10.5	18.2	19.00	9.30	52.11	.01962	.06671
〔11〕	0	18.2	11.39	5.77	31.24	.02030	.06688
	1	18.9	11.95	5.92	32.64	.01986	.06642

水となるまでにカルシウムが種々な変化を伴いつつ徐々に混合されてゆくことを示すものと思われる．これはまた内湾の安定な水塊に至るまでに塩素の混合よりもカルシウムの混合がおくれることを表はしているか又は塩素に対してカルシウムの増え方が次第に減少してゆくことを意味するものとみられ湾内で安定な海水となれば塩素比の値も一定となる．またこの曲線に於て塩素度 6～7 附近では，Ca/Cl のばらつきが大きくなつていることも，混合の不安定なことをあらわしているものと思はれる．これらを現在までに測定した落潮張

Table 3. (June 20, 1957)

St.	Depth. m	Temp. C°	Cl. g/l	Ca mg-atom/L	Mg mg-atom/L	Ca/Cl	Mg/Cl
〔1〕	0		3.44	2.04	9.70	0.02377	0.06858
	0.5	23.52	12.81	7.08	35.32	.02215	.06706
〔2〕	0		1.44	1.00	4.15	.02783	.07009
〔3〕	0	23.30	2.05	1.34	5.85	.02620	.06940
〔4〕	0	23.64	3.42	2.16	9.85	.02531	.07004
〔5〕	0	25.50	10.06	5.60	27.61	.02331	.06675
〔6〕	0	26.10	8.04	4.57	22.12	.02278	.06691
	1	25.53	—	7.44	36.59	—	—
	2	25.45	9.51	5.33	26.26	.02246	.06715
〔7〕	0	26.80	6.97	3.96	19.20	.02277	.06699
	1	25.50	14.56	7.89	40.13	.02172	.06686
	2	25.20	16.34	8.84	44.92	.02168	.06686
	3	24.48	17.75	9.62	—	.02172	—
	4	24.40	18.25	9.84	—	.02161	—
	5	24.11	18.31	9.94	50.19	.02176	.06666
	6	24.14	18.55	10.07	50.92	.02176	.06676
〔8〕	0	27.40	12.46	6.90	34.32	.02220	.06698
	1	27.50	14.13	7.77	38.95	.02204	.06704
	1.5	26.38	15.01	8.22	41.21	.02195	.06677
〔9〕	0	26.79	10.77	6.10	29.40	.02270	.06659
	1	27.18	10.90	6.11	29.79	.02247	.06647
	2	25.92	14.39	7.88	39.63	.02195	.06698
	3	24.65	17.97	9.72	49.26	.02168	.06667
	4	24.41	18.30	9.86	50.14	.02160	.06663
	5	24.44	18.46	9.95	50.47	.02159	.06649
〔10〕	0	27.64	11.31	6.33	31.06	.02243	.06679
	1	26.56	12.26	6.55	32.18	.02174	.06383
	2	25.90	12.93	7.16	35.63	.02219	.06702
〔11〕	0	28.03	11.09	6.21	30.53	.02244	.06695
	1	27.58	11.87	6.63	32.50	.02239	.06658
	2	25.97	16.19	8.85	44.41	.02190	.06683
〔12〕	0	25.20	16.76	—	—	—	—
	2	24.99	18.44	9.92	50.48	.02156	.06658
	10	24.50	18.29	9.86	49.92	.02161	.06638
	20	24.41	17.79	9.56	48.72	.02154	.06660
〔13〕	0	24.50	19.04	10.25	52.18	.02158	.06665
〔14〕	0	24.61	19.15	10.33	52.51	.02162	
	1	25.20	18.37	9.84	51.14	.02147	
	2	24.67	19.10	10.09	50.96	.02117	
	6	24.21	19.34	10.45	53.41	.02166	

潮の条件に於て比較すると第1図 A 及び B 曲線の如く低かんな水から高かんな外海水までに至る塩素比の変動のしかたは同様であるが張潮の場合

は比の値が高く安定した内湾水では殆んど外海水に近い値である．落潮では逆に塩素比は小さくなる．これは要するに外海水でのカルシウムの塩素比の値は第1表に示した如く 0.02124 で殆んど一定に近いから，これら塩素比の海況による差異は塩素量に比較してカルシウムの混合のしかたが遅いために生ずる差であるが或は潮汐に伴う水深の変化によつて底質からの直接又は間接的に与えるカルシウムの影響のあらわれであるかも知れない．現在までの測定では類以的な海況下ではいつも同様な結果が得られたが然しながら一日間の潮汐によつても海況差が生ずるわけであるから曲線 A と B の中間などの値も測定される筈であるが今迄の測定ではカルシウムの塩素比の差異は顕著でなかつためにこの海況差による塩素比の差異は比較的長い周期（例えば大潮と小潮の差異の如く）でくり返されるものの様である．これらのことについては今後尚検討する考えである。尚カルシウムと塩素度の関係は第2図に示す．

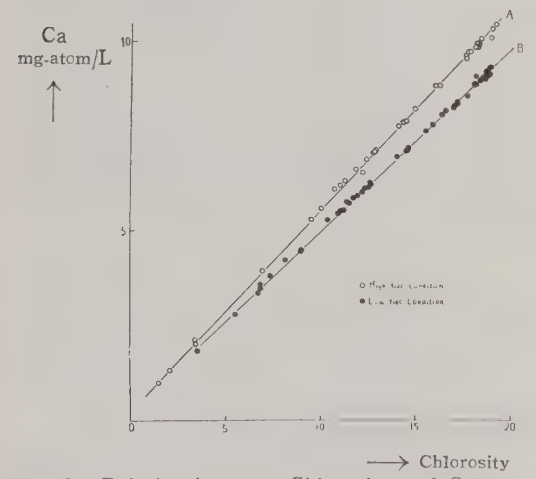

Fig. 2. Relation between Chlorosity and Calcium.

4. マグネシウムの塩素比について

カルシウムと同様にマグネシウムの塩素比を図示すると第3図の如くなる．これよりマグネシウムはカルシウムに比して顕著な変動がみられないことが判る．又前記カルシウムにみられる様な海況による差異もみられず浦戸浦では塩素量の変化にもかかわらず略一定である．尚マグネシウムと塩素度の関係は第4図に示す．

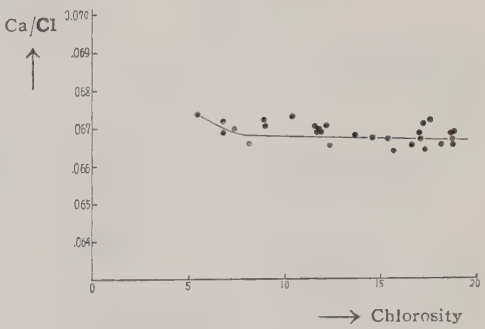

Fig. 3. Relation between Chlorosity and Mg/Cl.

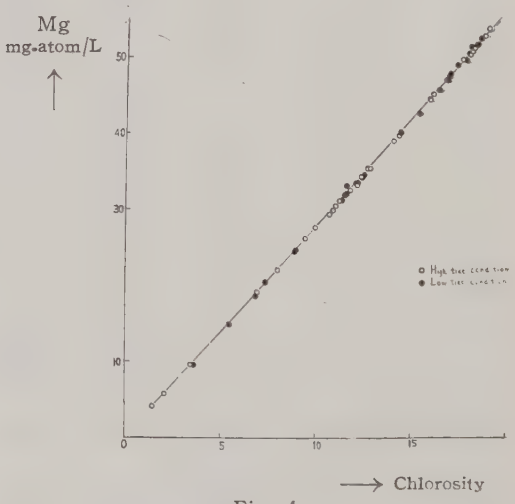

Fig. 4.
Relation between Chlorosity and Magnesium.

5. 結 語

以上のことから異水系の水が混合する際にはその組成が異りしかも化学元素の種類によつて混合の状態が異なること及びカルシウムでは同じ塩素量であつても海況条件によつてその存在量が異ることを明かにした．

終りに本研究に終始御懇篤な御指導を賜わつた京都大学有馬雅義教授に深甚な謝意を表すと共に御協力下さつた当教室今井嘉彦氏に感謝の意を表する．

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本邦内湾の海洋化学的研究 (第4報), 塩類組成の変動と海況との関係, (その2. 須崎湾)*

山　本　広　志**

Oceanographical Studies on Japanese Inlets (4), Relations between Variation of Chemical Compositions and Hydrographical Condition in Susaki Bay (Part 2.)

Hiroshi YAMAMOTO

Abstract: The author has observed the distribution of chlorinity and of water temperature and the chlorinity ratios of calcium and magnesium in Susaki Bay.

The surface water of this bay is diluted by land water, with a chlorinity of 15～16 Cl‰; while the chlorinity of the bottom water is equal to that of the open sea water.

In stable inlet water where the chlorinity is more than 17 Cl‰, the average chlorinity ratio of magnesium is 0.06647 and that of calcium is 0.02140, the latter is a slightly higher value than in the sea water. The variation in the chlorinity ratio caused by differences in hydrographic conditions is slight in this bay.

1. 緒　言

前報で指摘した通り，内湾に於けるカルシウム及びマグネシウムの塩素比は海況によつて特徴があるので．海況条件を異にする内湾でこれら化学元素の塩素比と海況との関係を検討すれば，内湾の特性についてなんらかの知見を得ることが出来るのではないかと考え外洋性の内湾であると思われる須崎湾について前報と同様な方法で研究を行つた．

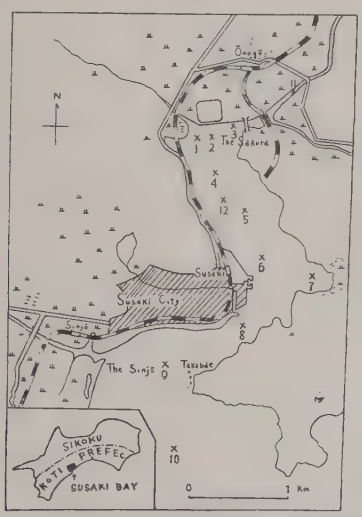

Fig. 1. Observational stations in Susaki Bay.

2. 須崎湾の地形的概況

本湾は湾口が広く土佐湾に面する小規模な内湾で面積約 1.28 km² の内湾と，土佐湾の沿岸の一部と見做される外湾とより成り，外湾は全く外海性である．本湾に流入する河川は内湾に桜川，外湾に新荘川があつて表層の塩分濃度を低下させる原因となつている．水深は内湾の中央部で 9 m，狭小部で 16 m，外湾は 12～18 m 程度である．

3. 実験結果及び考察

a. 水温及び塩素量の分布

本湾の塩素量は表層で，外洋水よりは僅かに低かんで 15～16 Cl‰ である．これは流入河川水の影響と考えられ，この低かんな水は 3 m 以深に及ぶことはない．1957年10月16日の調査資料にもとづいて水温及び塩素量の分布を第2図に挙げた．採水点は第1図の通りである．これによると表層に低温低かんな水塊があり，湾奥は比較的高温高かんで，外海に近い水域では低温低かんとな

* 1958年7月30日受理.

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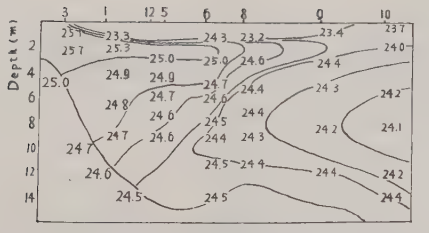

Fig. 2-a. Sectional distribution of temperature in Sts. (Oct. 16, 1957).

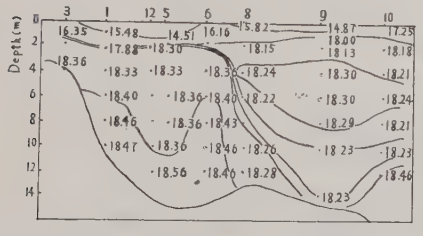

Fig. 2-b. Sectional distribution of chlorinity in Sts. (Oct. 16, 1957)

つている．この外海に近い水域での低温低かんは新荘川の影響によるものであろう．

b. カルシウム及びマグネシウムの塩素比とその変動

第1図に示した採水点で種々な深さでカルシウム及びマグネシウムの塩素比を測定した．その結果は第1表及び第3図に示した．

この図からも明らかな如く，前報で述べた浦戸湾と同様に海水の塩素量が陸水に稀釈されて低下すると，カルシウム及びマグネシウムの塩素比は漸時増加する傾向を示し，湾内の安定した水塊で

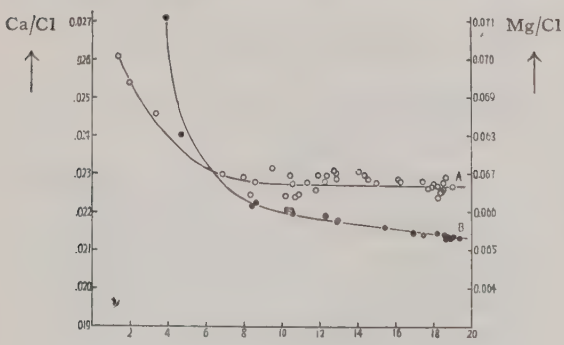

Fig. 3. A: Relation between Chlorosity and Mg/Cl.
B: Relation between Chlorosity and Ca/Cl.

Table 1.

St.	Depth. m	Temp. C°	Cl. g/l	Ca mg-atom/L	Mg mg-atom/L	Ca/Cl	Mg/Cl
〔1〕	0	24.5	4.72	2.83	12.90	0.02403	0.06647
	0.75	24.2	9.51	5.32	26.20	.02242	.06700
〔2〕	0	23.6	8.68	4.82	23.84	.02226	.06680
	1	23.9	8.44	4.67	23.06	.02218	.06648
	2	23.4	17.48	9.35	48.01	.02144	.06680
〔3〕	0	23.6	10.51	5.79	23.93	.02208	.06694
	0.5	23.6	10.25	5.65	27.96	.02209	.06639
	1	—	12.92	7.02	35.59	.02178	.06693
〔4〕	0	23.8	12.32	6.74	33.84	.02193	.06680
	1	23.7	12.97	7.08	35.67	.02179	.06688
	2	22.2	18.20	9.74	49.93	.02145	.06672
	3	21.7	18.53	9.93	50.73	.02148	.06658
	4	21.5	18.66	10.00	51.18	.02149	.06670
〔6〕	0	21.8	16.96	9.09	46.34	.02148	.06645
	2	21.9	18.34	9.84	50.00	.02150	.06630
	4	21.3	18.81	10.00	51.33	.02131	.06636
	6	21.1	18.91	10.09	51.56	.02139	.06631
	8	20.7	19.19	10.21	52.60	.02132	.06666
	10	20.4	19.30	10,28	52.39	.02135	.06601
	12	20.3	19.36	10.30	52.81	.02132	.06634
	14	20.1	19.31	10.31	52.65		.06631
	15.8	19.9	19.48	10.32	53.00	.02123	.06617
〔7〕	0	23.7	15.44	8.33	42.32	.02162	.06666
	2	21.5	18.70	10.02	51.36	.02148	.06680
〔11〕		—	0.11	0.39	0.42	.14210	.09286

17 Cl‰ 以上の場合については平均 Mg/Cl は 0.06647, Ca/Cl は 0.02140 で外海水に比較するとカルシウムの場合が僅かに高い結果が得られた．両成分共塩素比が変動してゆく傾向はこのように浦戸湾に類似しているが，海況の差異による塩素比の変動は大きな差が認められなかつた．このことは浦戸湾に比較して陸水の影響によつて組成が不安定となつた状態が比較的表層附近に限られ，下層は安定な水塊で占められていると考へられる．

なおカルシウム及びマグネシウムと塩素量の関係は第4図に示す如く直線上に分布する．

市栄[1]等は海水に河川水等の陸水が影響する場合の関係について，流入河川水の水量が重要な要素となり，垂直的にみれば3m以深は間接的な影響であるとしている．同様に須崎湾が浦戸湾に比べ

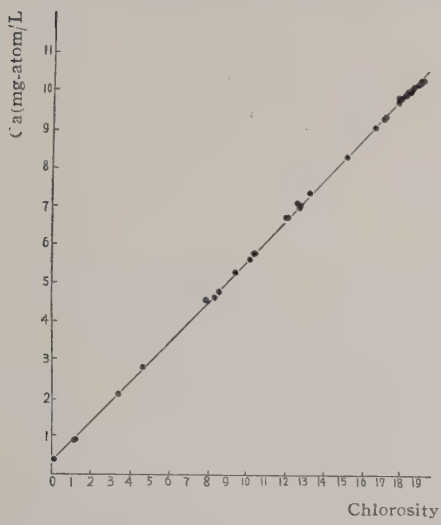

Fig. 4-a. Relation between Chlorosity and Calcium.

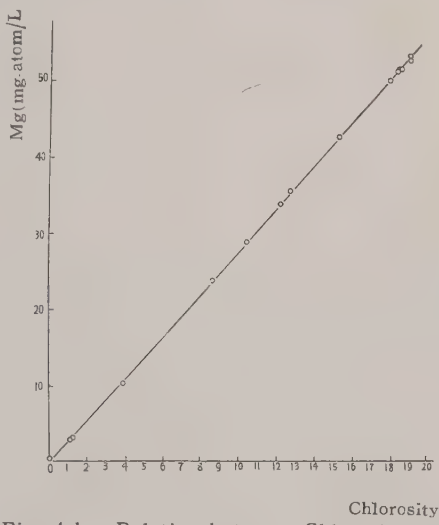

Fig. 4-b. Relation between Chlorosity and Magnesium.

て塩素比が高いことは浦戸湾よりも水量が少いことと，今一つの条件として水量と共に陸水中のカルシウムの多少によつても塩素比の変動のしかたが異なつてくると思われるので，桜川のカルシウムの存在量が浦戸湾に流入する各河川の存在量よりも大きいことから恐らくそれら両者による影響であろうと考へられる．

またCa/Clの極大値が浦戸湾よりも高い塩素度にあることは，この流入河川中のカルシウムが多いことを示している．このように両湾の如く陸水によつて稀釈されるような内湾水では稀釈されるにつれてCa/Cl及びMg/Clの値は塩素量が6～7‰附近までは徐々にその後は急激に上昇する．

尚内湾全般についてのカルシウム及びマグネシウムの塩素比と海況との関係についての総括的な考察は次報に於て述べる．

終りに臨み本研究に御懇篤な御指導を賜わつた京都大学石橋雅義教授に深甚の謝意を表すると共に御協力下さつた当教室の今井嘉彦氏並に小林光春君に感謝の意を表する．

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寄　稿

カナダ通信*

宇田道隆**

小生出発前学会で歓送頂きありがとうございました．大圏コースを長航，50°N をこえ1日3回表面観測しながら，自ら海の色，温度，気象を体験，早速今の研究に役立ち東太平洋の極前線(?)をPOGで議論するデーターになりました．（僕が船で観測しながら来たことが皆から大へん好感もたれ，タリーも今度日本え行くときは船にするといっています．）9月13日シアトル着．ワシントン大学海洋学部長 R.H. Fleming 教授と滞在中の名大教授(化)小山忠四郎夫妻のお出迎えをうけました．(T. G. Thompson 老先生は，マッコーネル島避暑で会えず．) 14日(日) ワシントン大学構内を見学．海洋学部 Brown Bear 号(塩分は電導度測定の Salinometer だけで出しています，C^{14} もやっています，IGY で 5000 m. まで観測の由）をみました．15日から Puget Sound 観測に出動，塩分分布，潮汐流による混合が主題で，ここの Lincoln 氏が大規模な模型実験を今やつてるのを比較する由．小山さんは海水中の有機物（蟻酸，醋酸等）の分析研究され，内湾表層に多いという結果が出たと話されました．広大壮麗完備した大学 (U. W.)，その景色の美しく，草花と木々建築に讃歎せざるを得ませんでした．シアトルは British Columbia 共今年百年祭で大にぎわいですが，ワシントン大学は1861年創立ですから，開拓者の偉大な先見に驚きます．北米は今年は異例の乾燥した暑夏で，いくら水をやつても大学の青芝生も枯れた由，フレミングさんは「沿岸水温も数度高かつた」といい，後でカナダできくと，B. C. でも同様で，5月～9月水温2°C以上平年より高く，南方系の珍らしい魚がたくさんとれたり，サケ（当地方魚の王様です）の来游遡河が2旬以上おくれ，回游水層も（刺網により）表層高温をさけていつもより深く，回游路も大変化して沖合暖水帯におされて極めて接岸，空前の記録的大漁年(昨年の2倍，紅ザケ特に大漁，ピンクも大漁）クジラ（バンクーバー島基地）今までにない（例年 400～500 頭が今年は 774 頭）大漁で，組成も著変し，ナガスが主で，大形マッコー少く，やはり暖水帯の圧迫で，事業場近くの距岸数十マイルに塊つた好条件のせいとされています．California 海流系のジョー・リード等の報告でも1957年暖化，イワシ漁好転とあり，北洋日本と今年の異変は注目すべき研究問題です．15日バンクーバーで笠原昇博士（日米加北太平洋漁業会議事務局駐在）に車で迎えられ，渡船場ではナナイモの所長 Needler 博士らにあい，夜ナナイモ着港，太平洋海洋学研究所長タリー博士に迎えられホテルへ．彼の自宅のお茶によばれ，夫人にお目にかかり，到着をワインの盃で祝いました．それから今日まで約20日間です．人口 2 万の実にきれいな可愛い平和な町です．研究所は4マイル離れた Departure Bay 海岸の実に美しい明るい風景の場所です．海岸にはニシンカモメが鳴きあそび，2～3月この磯へニシンが産卵に群来する由．このすぐ沖で曳縄，まき網，刺網で，この間までサケ漁でにぎわいました．ここの所員ものんきで，休日は今解禁の鹿射ちに裏山へ行つたり，サケ釣を楽しんだりです．うつそうとした緑林にかこまれ，駒鳥など多く，夕方野鳩がとんでいます．小生は見晴しのよい日当りもとてもよい立派な1室を頂きました．今，Scripps のマグロ研に行つておる E. P. Bennett (NORPAC のカナダのデータをまとめた，物理）が居た由．勤務は 8.30～4.45きつかりで，皆時間は実に正しく守ります．お茶が10時と2時半に15分，昼12時から45分休み，談論風発実にたのしみです．みんな親切な好い人ぞろいで，僕が眼鏡のツルがとれて弱つていると，すぐ工作室え持つていつて直してくれたり，ぬりたてのペンキを知らずに服へつけると，ベンゼンでふきとつてくれたり，次次，土日に招待を受け，ピクニックはカーで連れてくれ，感謝ばかり．日本でこんなに皆でしてあげれたらと思います．小生の従来の経験範囲では，研究所の雰囲気設備は極上で，学ぶべき点がとても多く，仕事はすばらしく能率的でムダなく，小生もとても忙しく，これまでにない充実した研究生活をもつ事が出来，頭の中も雑事に悩されないためか実に爽快で，人間的にも周囲から日々教えられるところが多く，本当に来てよかつたと喜んでいます．少し人間が改善されて帰れると思います．お笑い下さい．観測へ出かけた人はすつかりデータを整理して来てすぐ印刷にかけ，今夏（7. 8月）の北洋大観測成果も1ヶ月后もう本になつており，POG の Dodimead さん（とてもこの上ない好人物です）が10月中旬東京の（北大平洋漁業）会議へ出ます．（タリーはこの次といつています）．Biol. st. から所長 Neddler や Neave, Manglis (寄生虫からアジア系，アメリカ系サケの分布図をつくりました）等行きます．Dodinead とは，きのうも，そのまとめられた結果を討論しました．ここは暮しよい気候も実によい所で，真冬も東京より暖いようです．オーバー持参しましたが，先ず不用のようです．お昼は持参のサンドイッチ，果物，ケーキ入の弁当と，コーヒー茶わん持つて海岸の日当りよいところに並び，ダベりながらパクつき，全く毎日がピクニック気分です．しかし仕事にかけては実に真剣で，1957. 58年と冬季困難な観測をアリューシャン，ベーリング方面（アラスカ湾は勿論）やり，春夏秋と続け，大調査船日下計画中の由，水産も海洋も研究所は，ここ数年に倍以上膨脹躍進をたどり，実に大拡

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張の計画が出ており，全て綜合的に行われ，海洋も気象，水路，海軍，大学，漁業1本で（今タリーとニードラーが9月30日～10月6日オッタワのその協議会出席中）進められ，ここの POG と漁業の協力も実にすばらしく，Strickland（ここの海洋生物学者）がいつたように皆，の人がいつも共通連帯の問題を頭に画いてやつていることがわかります．

アメリカのスクリップス，Texas もそうでしよう．小生 Tully らと10日間討論，黒板前にありたけの知識をサラケ出してどなるようにやりました后今研究のスタートを切つています．中冷層 (dicothermal layer)，中暖層 (mesothermal)，塩分躍層 (halocline で Salinocline)，Entrainment（つれこみ）等，天鷹丸の今夏，平野敏行さん忍路丸の成果と Dodimead さんのと合すと面白いでしよう．Sub-Arctic Pacific の Oceanography が第1．その漁業への応用（サケが外海の何処におるか，海洋生活生残，接岸海況など）第2．の要望による第2部報告．の外，所内で今月末から20回講義，毎週(金)午后3時より小生40分話し，(予めタイプしたノートを配布—3・4頁毎回—しておく) 30分討論，10回は海況学（東大・理・大学院でやつている分）あと10回は，(北大漁場学講義の分に基く分）スライド図使用，文献展示，日本の方々の業績紹介します．所内の distinguished scientist（海洋，水産）相手に英語の討論なので小生も大ハリキリです．今毎日英文ノート作り，此の同じ講義を U. W.(シアトル．ワシントン大学)，U.B.C.（ブリテッシュ．コロンビヤ大学）で1，2月ごろやります．U.B.C.（バンクーバー）へは，4月29日訪，海洋学研前所長 Dr. Clemens（目下「カナダの魚学」大著増訂中），現所長 G. L. Pickhard 教授（物理，沿岸水問題研究中），Stewart 教授（数，Reynolds stress 等）・となりの漁業研究所長 Larkin（淡水生物）不在で Lindsey（魚の Taxonomy）Scagel（海藻）に再会，collection 見せてもらい，日本より留学中の淡水研 三浦，上野氏に会いました．（セイロンからも留学しています．）Pickard が car で広大な十数哩もある大学 campus を案内，原始林をひらいてビルがぞくぞく建設中，牧場もあり，美しく未来の発展希望にみちみちています．お宅で夫人，令嬢，母堂に会い色々珍しいものみせてもらいました．当 POG 内には，北氷洋調査部門（軍関係）から沿岸水路関係もふくまれ，300 マイル以内沿海観測定地沿岸観測（日々暨台船2隻含む）定点観測（研究部）を包含．研究室は，1.2 階に20室位．地下に工作室，検定室（BT 検定機はここ特製の注目すべきもの），測器調整室，試験室あり．事務，印刷，図書の大きな分は，本館に水産生物と共有，大金庫に帰りに大切な原稿資料しまい，大きな図は，ケース別にあり，小生ももらいました．海洋物理に Fofonoff 博士が海況予報研究と模型実験など理論的指導に当り，俊才で白系ロ人なので露文海洋文献小生にもきのう1958のソ連新刊海洋学研究報告，図の解説書いてくれ実に親切感謝しています．田畑進（2世）も U.B.C. 卒で定点海洋気象（日射，光等）担当，熱経済にとりくみ，化学で生産力の Strickland 博士（若手 McAllister, Le Brasseur, プランクトン）に協力しています．この基礎生産力の仕事が，昨年からはじまり，今にすばらしい成果を生むでしよう．この連中が特に親しくしています．Dodimead, Barber, Hollister 等実地海況連中は勿論毎日いつしよです．最も注目すべきは Tully, Fofonoff の完成した Hecate Strait の大模型実験室が丘上に出来た事です．数十メートルの経で高さ数十メートルの柱の上にワイヤ張り，synchronized camera でうつす．electronic device で潮汐流，陸水流入，外洋水進入をおこし，大鮮の卵稚魚の撒布や汚染問題やるらしく，工作工場が隣にあり，実験海洋学として最も興味もたれます．ここの塩分は全部 salinometer（電橋式3サーモスタット，15分で6サンプル処理）で，ここ特製といつていますが U. W. のと同じです．タリー考案のベロウから自記各層流速計2年前から Test 改良中です．Brett，サケの大飼育生態実験水槽進めています．

特別寄稿

潮汐理論* (第2回)

日高　孝次**

II. 自転する地球上の潮汐

1. 潮汐運動の微分方程式　水の振動が回転する円板上に起ると考えられる，即ち $\omega \sin\varphi = -$定．の場合は，当然その結果は $\sin\varphi$ が一定である様な海でなければ厳密には正しくない．地球上でその解が近似的にでも適用され得るのは南北極附近であるがそれでは地球全体の潮汐を論ずるには役立たない．後者の場合にはどうしても地球を少くも自転する球であるとし，その表面を覆う海を考えなければならない．

攪乱天体の影響がなく海面が静止している場合には自由水面は重力及び遠心力に対し平衡状態にある．今地球の自由海面は楕円率 (ellipticity) の小さい廻転楕円体の表面と一致していると考える．

かかる平衡自由水面を基準水面に採り其上の1点に於て θ を余緯度 (co-latitude) 即ち自由海面に立てた法線

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が地球の極軸となす角，λ を経度とし，また z を自由海面への法線に沿うて外方正に採った水面上の高さとする．

斯の如く地球に対する水粒子の位置を互に直交する座標 θ, λ, 及び z で表す場合には単位質量のもつ運動勢力 T は

$$2T=(R+z)^2\dot{\theta}^2+\varpi^2(\omega+\dot{\lambda})^2+\dot{z}^2 \qquad (1.1)$$

で与えられる．茲に R は規準水面を子午線面で截つた曲線上の曲率半径，ϖ^2 は水粒子から極軸に至る距離である．R は θ 丈けの函数で大体地球の平均半径に等しく ϖ は θ 及び z の函数である．また簡単な幾何学的の計算で

$$\frac{\partial\varpi}{(R+z)\partial\theta}=\cos\theta, \qquad \frac{\partial\varpi}{\partial z}=\sin\theta \qquad (1.2)$$

であることが判る．

そこで Lagrange の広義座標に関する運動の方程式の左辺に依って加速度の成分を計算すると，θ, λ, z の方向について各々

$$\frac{d}{dt}\left\{\frac{\partial T}{(R+z)\partial\dot{\theta}}\right\}-\frac{\partial T}{(R+z)\partial\theta}, \quad \frac{d}{dt}\left(\frac{\partial T}{\varpi\partial\dot{\lambda}}\right)-\frac{\partial T}{\varpi\partial\lambda}$$

及び $\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial\dot{z}}\right)-\dfrac{\partial T}{\partial z}$ であるから運動を無限に小さいものとして小さい量を省略すると成分は各々

$$\left.\begin{aligned}
&\frac{1}{R+z}\left(\frac{d}{dt}\frac{\partial T}{\partial\dot{\theta}}-\frac{\partial T}{\partial\theta}\right)\\
&\quad=(R+z)\ddot{\theta}-\frac{1}{R+z}(\omega^2+2\omega\dot{\lambda})\varpi\frac{\partial\varpi}{\partial\theta},\\
&\frac{1}{\varpi}\left(\frac{d}{dt}\frac{\partial T}{\partial\dot{\lambda}}-\frac{\partial T}{\partial\lambda}\right)\\
&\quad=\varpi\ddot{\lambda}+2\omega\left(\frac{\partial\varpi}{\partial\theta}\dot{\theta}+\frac{\partial\varpi}{\partial z}\dot{z}\right),\\
&\frac{d}{dt}\frac{\partial T}{\partial\dot{z}}-\frac{\partial T}{\partial z}=\ddot{z}-(\omega^2+2\omega\lambda)\varpi\frac{\partial\varpi}{\partial Z}
\end{aligned}\right\} \qquad (1.3)$$

となる．故に今地球に対する θ, λ 及び鉛直方向の水分子の速度成分を夫々 u, v, w とすると

$$u=(R+z)\dot{\theta}, \quad v=\varpi\dot{\lambda}, \quad w=\dot{z} \qquad (1.4)$$

であるから (1.2) を注意し乍ら (1.3) を変形し流体力学の運動の方程式に代入すると次の様になる．

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}-2\omega v\cos\theta\\
&\quad=-\frac{1}{R+z}\cdot\frac{\partial}{\partial}\left(\frac{p}{\rho}+\Psi-\frac{1}{2}\omega^2\varpi^2+\Omega\right),\\
&\frac{\partial v}{\partial t}+2\omega u\cos\theta+2\omega w\sin\theta\\
&\quad=-\frac{1}{\varpi}\frac{\partial}{\partial\lambda}\left(\frac{p}{\rho}+\Psi-\omega^2\varpi^2+\Omega\right).\\
&\frac{\partial w}{\partial t}-2\omega v\sin\theta=-\frac{\partial}{\partial z}\left(\frac{p}{\rho}+\Psi-\frac{1}{2}\omega^2\varpi^2+\Omega\right).
\end{aligned}\right\} \qquad (1.5)$$

茲に Ψ は地球の重力ポテンシァル，Ω は太陰及び太陽の攪乱力即ち起潮力のポテンシァルである．

今迄の議論で省略したものは u, v, w について第二位の量丈けである．併し地球の半径に較べると海の水深は小さいから (1.5) の $R+z$ の代りに R と書いても一向に差支ない．更に鉛直速度 w は水平速度 u, v に対して小さいと考へて宜しいから ωv に対して $\partial w/\partial t$ を省略することも差支ない．かゝる省略はその結果と矛盾しない限り「長波」の理論で許容し得ること勿論である．かくの如くして (1.5) の第3式は

$$2\omega v\sin\theta=\frac{\partial}{\partial z}\left(\frac{p}{\rho}+\Psi-\frac{1}{2}\omega^2\varpi^2+\Omega\right) \qquad (1.6)$$

となる．

そこで次の工作に移る．水面が全然攪乱されてない時この面を $z=0$ と採るならばこの面では $\Psi-\dfrac{1}{2}\omega^2\varpi^2$ は一定であるから

$$z=0: \quad \Psi-\frac{1}{2}\omega^2\varpi^2=\text{一定}$$

である．然るに実際は水面が $z=0$ よりも高かったり低かったりする．今攪乱された実際の水面は $z=\zeta$ であるとすると

$$\begin{aligned}
&\left[\Psi-\frac{1}{2}\omega^2\varpi^2\right]_\zeta\\
&\quad=\left[\Psi-\frac{1}{2}\omega^2\varpi^2\right]_0+\left[\frac{\partial}{\partial z}\left(\Psi-\frac{1}{2}\omega^2\varpi^2\right)\right]_0\cdot\zeta+\cdots
\end{aligned}$$

であるから $z=0$ なる面に於る重力の見掛上の値を g とすると $g=\left[\dfrac{\partial}{\partial z}\left(\Psi-\dfrac{1}{2}\omega^2\varpi^2\right)\right]_0$ となる．即ち $z=\zeta$ では

$$z=\zeta: \Psi-\frac{1}{2}\omega^2\varpi^2=g\zeta+\text{常数} \qquad (1.7)$$

となる．今 (1.6) を z から ζ 迄積分すると

$$\begin{aligned}
&\left[\frac{p}{\rho}+\Psi-\frac{1}{2}\omega^2\varpi^2+\Omega\right]_z\\
&\quad=\left[\frac{p}{\rho}+\Psi-\frac{1}{2}\omega^2\varpi^2+\Omega\right]_\zeta-2\omega\sin\theta\int_z^\zeta v dz
\end{aligned}$$

となるが $z=\zeta$ では圧力が一定値に等しく，また攪乱力ポテンシァル Ω は殆ど z に関係しない事を考慮し，且つ (1.7) を注意すれば任意の高さ z では

$$\frac{p}{\rho}+\Psi-\frac{1}{2}\omega^2\varpi^2=g\zeta-2\omega\sin\theta\int_z^\zeta v dz+\text{常数} \qquad (1.8)$$

である．また最後の項は海の水深を h とすると大体 $\omega hv\sin\theta$ 程度の量であるが，これと $g\zeta$ との比は大体 h/a に等しいから省略しても宜しい*．そこで (1.8) を (1.5) の第1，第2両式に代入すれば

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t}-2\omega v\cos\theta=-\frac{g\partial}{R\partial\theta}(\zeta-\bar{\zeta}),\\
&\frac{\partial v}{\partial t}+2\omega u\cos\theta=-\frac{g\partial}{\varpi\partial\lambda}(\zeta-\bar{\zeta})
\end{aligned}\right\} \qquad (1.9)$$

となる．但し $\bar{\zeta}$ は平衡潮高で

$$\bar{\zeta}=-\Omega/g \qquad (1.10)$$

で与えられることは既に平衡潮汐論に述べた通りである．(1.9) は z を含んでいないから潮汐運動では水平方向の運動は同一鉛直面では略々同一と考えることができる．

次に連続の条件は底面積 $\varpi\delta\lambda\cdot R\delta\theta$，高さ h の柱状空間から δt なる時間中に流出する水の量が同じ時間内に於ける水面の下降 $\delta\zeta$ に伴ひ同じ空間で水が失はれる量に等しいとすると

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* 次の (1.9) の両辺に h を乗じ $R\sim a$ としてみると $\omega hv\cos\theta$ と $\dfrac{h}{a}\dfrac{\partial}{\partial\theta}(g\zeta)$ とが同位である．依って $\omega kv\sin\theta$ と $g\zeta$ との比は大凡 h/a に等しい．

$$\delta t\left\{\frac{\partial}{\partial\theta}(uh\varpi\delta\lambda)\delta\theta+\frac{\partial}{\partial\lambda}(vhR\delta\theta)\delta\lambda\right\}=-\varpi\delta\lambda\cdot R\delta\theta\cdot\delta\zeta$$

なる式が成立つから連続の式は次の様になる．

$$\frac{\partial\zeta}{\partial t}=-\frac{1}{\varpi}\left\{\frac{\partial(\varpi hu)}{R\partial\theta}+\frac{\partial(hv)}{\partial\lambda}\right\}\qquad(1.11)$$

かくして得た方程式 (1.9) (1.11) には以上の仮定以外には別に仮定を含まない．特に地球が有つ楕円率については何の拘束も考える必要がないのである．併し差支ない範囲でできるだけ問題を簡単にする為にこの楕円率が略 $\frac{1}{289}$ で大体遠心力と重力との比即ち ω^2a/g に等しいことを考慮し，この程度の誤差を許容すれば

$$R=a,\quad \varpi=a\sin\theta,\quad g=\text{常数}$$

として宜しい．但し a は地球の平均半径である．かくの如くして潮汐運動の方程式は

$$\left.\begin{aligned}\frac{\partial u}{\partial t}-2\omega v\cos\theta&=-\frac{g}{a}\frac{\partial}{\partial\theta}(\zeta-\bar{\zeta}),\\ \frac{\partial u}{\partial t}+2\omega u\cos\theta&=-\frac{1}{a\sin\theta}\frac{\partial}{\partial\lambda}(\zeta-\bar{\zeta}),\\ \frac{\partial\zeta}{\partial t}&=-\frac{1}{a\sin\theta}\left\{\frac{\partial(hu\sin\theta)}{\partial\theta}+\frac{\partial(hv)}{\partial\lambda}\right\},\end{aligned}\right\}(1.12)$$

となる．

2. Laplace の与えた3種の潮汐　さて Laplace は以上の微分方程式を解いて地球上の海洋に起る潮汐現象を解決しようとしたが，まづ起潮力のポテンシャル $\bar{\zeta}$ が

$$\begin{aligned}\bar{\zeta}&=\frac{3}{2}H(\cos^2\Delta-\frac{1}{3})(\cos^2\theta-\frac{1}{3})\\&+\frac{1}{2}H\sin 2\Delta\sin 2\theta\cos(\alpha+\lambda)\\&+\frac{1}{2}H\sin^2\Delta\sin^2\theta\cos(2\alpha+2\lambda)+C\qquad(2.1)\end{aligned}$$

の如く展開できることを示した．茲に Δ は北極から起算した天体の距離角，α は天体の時角，λ は緯度で H は

$$H=\frac{3}{2}\cdot\frac{M}{E}\left(\frac{a}{D}\right)^3\cdot a\qquad(2.2)$$

で与えられる．但し M 及び E は各々攪乱天体及び地球の質量，D は天体の中心から地球の中心に至る距離である．

扨て (2.1) に依って見ると θ は地球上の一点では一定であるが Δ は太陰で約29日，太陽では1年の週期で変化するから

$$\cos^2\Delta-\frac{1}{3},\ \sin^2\Delta,\ \sin 2\Delta$$

等の量はその半分，即ち太陰では約半月，太陽では半年の週期で変化する．併し乍らこれらの量は時間について平均してみると決して零にならない．

Laplace は以上の理由に基き天体の引力に依って海洋に起る可き強制振動即ち潮汐には3種の型があることを指適した．その1つは (2.1) の第1項で表される振動即ち「長週期潮汐」で，それに対応した平衡潮高は自転軸の周りに対称的，即ち経度に無関係な振動で「Laplace の第1種の振動 (Laplace's oscillations of the first species)」と呼ばれるものである．かゝる振動では $\cos^2\theta-\frac{1}{3}=0$ に相当する平行圏，即ち南北緯 35°16′ が節線となる．第2項は「一日潮汐」を与え，その平衡潮高は両極を通り地球を一周し攪乱天体の方向から 90° を距つる全子午線が1つの節線になり赤道が今1つの節線となる型の振動であり，これを「Laplace の第2種の振動 (Laplace's oscillations of the second species)」という．最後は即ち平衡潮高の分布が攪乱天体直下の子午線から 45° 及び 135° を距つる2つの全子午線のみを節線に有つ振動でこれを「Laplace の第3種の振動 (Laplace's oscillations of the third species)」といい「半日潮汐」に相当するものである．更に自由振動についても同じく第1種，第2種，第3種を区別することができる．この場合には第1種振動とは振動が極軸の周りに対称で経度に無関係なもの，第2種とは1本の全子午線 ($\lambda=0°,\lambda=180°$) が節線とが節線となるもの，第3種とは2本の全子午線が節線となるものである．

かくの如く起潮力は3つの項から成るが太陰及び太陽の運動が甚しく複雑である為に Δ も α も決して時間と正比例することなく，これらの週期も振幅も決して一定でない．従って起潮力も潮汐も大体週期的に変化する3つの項から成りはするが決して正しい3つの正弦運動から成るのではない．

併し乍ら Lord Keivin, G. H. Darwin, H. Rauschelbach や A. T. Doodson 等に依ってこれら大体週期的の項を幾つかの純粋に週期的の項の和に分解することが企てられ，その結果起潮力に依る平衡潮高 $\bar{\zeta}$ を

$$\zeta=\sum_\sigma H_0(\cos^2-\frac{1}{3})\cos(\sigma t+\varepsilon_0)\qquad(2.3)$$

なる「長週期」の項の和と

$$\zeta=\sum_\sigma H_1\sin\theta\cos\theta\cos(\sigma t+\lambda+\varepsilon_1)\qquad(2.4)$$

の如き「一日週期」の項の和及び

$$\bar{\zeta}=\sum_\sigma H_2\sin^2\theta\cos(\sigma t+2\lambda+\varepsilon_2)\qquad(2.5)$$

の如き「半日週期」の項の和とに別つことに成功した．このことは赤道の周りを正しい円形の軌道を描いて地球を公転する無数の天体が存在することと同等である．そうしてこれら各項に依って起る潮汐はこれを「分潮 (constituents; components)」と称し，起潮力の相当項と同一週期を有つもので，水位の昇降はそれら分潮の和として現れる．これで見ると各分潮は何れも純粋に週期的の項から成るがその振幅は θ の函数である．尚 H_0, H_1 及び H_2 は Darwin 等がこれを計算決定している既知常数である．

かくの如くして $\bar{\zeta}$ は3群の週期函数の和から成ることを見たがその和号下の週期は長週期潮汐では1年，半年，1ケ月，半月を週期とするものを含み，一日潮汐では太陰一日潮 O_1，日月合成一日潮 K_1，半日潮汐には太陰半日潮 M_2，太陽半日潮 S_2 等何れも1日及び半日と僅かに異る週期を有つ分潮の集合として表されるのである．これら分潮の週期については後に述べる．

3. 動力学的潮汐論　初め Laplace がこの問題を論じた時に彼は一日及び半日週期の振動では成程運動を動力学的に論ずる必要があるが，長週期潮汐では最短週期

のものでも14日余の週期を有つから，かゝる緩慢な振動では海水の摩擦に依り運動は動力学的の理論に従はず寧ろ時々刻々外力と釣合いその結果は略ゝ平衡潮汐論で得たものと等しくなると考えた．従ってLaplaceは長週期潮汐には動力学的の取扱いを適用しなかった．併しその後Kelvin*及びDarwin**は長週期潮汐と雖も必ずしも平衡説に従うものでないことを論じ，これを動力学的に取扱った．今その方法を次に述べる．

4. 長週期潮汐

長週期潮汐では水位は経度 λ と無関係であるから (1.12) の中で ζ, u, v が共に

$$e^{i(\sigma t+\varepsilon_0)}$$

なる因数を有つとする．また $\bar{\zeta}$ は

$$\bar{\zeta}=H_0(\cos^2\theta-\frac{1}{3})e^{i(\sigma t+\varepsilon_0)} \tag{4.1}$$

となる．依ってかくの如くして t に関する項を消去すると (1.12) は

$$\left.\begin{aligned} u&=\frac{i\sigma g}{\sigma^2-\omega^2\cos^2\theta}\cdot\frac{\partial}{a\partial\theta}(\zeta-\bar{\zeta}),\\ v&=-\frac{2\omega g\cos\theta}{\sigma^2-4\omega^2\cos^2\theta}\cdot\frac{\partial}{a\partial\theta}(\zeta-\bar{\zeta})\end{aligned}\right\} \tag{4.2}$$

$$i\sigma\zeta=\frac{\partial(hu\sin\theta)}{a\sin\theta\partial\theta} \tag{4.3}$$

となるから今

$$\frac{\sigma}{2\omega}=f,\qquad \cos\theta=\mu \tag{4.4}$$

と置いて (4.2) 及び (4.3) から u を消去し且つ海の水深 h を至る処一様とすれば

$$\frac{\partial}{\partial\mu}\left\{\frac{1-\mu^2}{f^2-\mu^2}\frac{\partial\zeta'}{\partial\mu}\right\}+\beta\zeta=0 \tag{4.5}$$

が得られる．但し $\zeta'=\zeta-\bar{\zeta}$ で

$$\beta=\frac{4\omega^2a^2}{gh} \tag{4.6}$$

である．玆に仮定に依り f は1に対して極めて小さい．

さて $\bar{\zeta}$ に於て t の項を落し $\cos\theta=\mu$ と置けば

$$\bar{\zeta}=H_0(\mu^2-\frac{1}{3}) \tag{4.7}$$

となる．従って微分方程式は

$$\frac{\partial}{\partial\mu}\left(\frac{1-\mu^2}{f^2-\mu^2}\frac{\partial\zeta'}{\partial\mu}\right)+\beta\zeta'=-\beta H_0(\mu^2-\frac{1}{3}) \tag{4.8}$$

となる．ζ の値は両極 ($\mu=\pm1$) では $+\frac{2}{3}H_0$ となり赤道上 ($\mu=0$) では $-\frac{1}{3}H_0$ で与えられることは (4.7) から明瞭な通りである．(4.8) を積分するために B_1, B_3, …を常数として

$$\frac{1}{f^2-\mu^2}\frac{\partial\zeta'}{\partial\mu}=B_1\mu+B_3\mu^3+\cdots+B_{2j+1}\mu^{2j+1}+\cdots \tag{4.9}$$

と置けば

$$\frac{\partial}{\partial\mu}\left(\frac{1-\mu^2}{f^2-\mu^2}\frac{\partial\zeta'}{\partial\mu}\right)=B_1+3(B_3-B_1)\mu^2+\cdots +(2j+1)(B_{2j+1}-B_{2j-1})\mu^{2j}+\cdots \tag{4.10}$$

$$\zeta'=A+\frac{1}{2}f^2B_1\mu^2+\frac{1}{4}(f^2B_3-B_1)\mu^4+\cdots +\frac{1}{2j}(f^2B_{2j-1}-B_{2j-3})\mu^{2j}+\cdots \tag{4.11}$$

となる．但し A は積分常数である．依って (4.10) 及び (4.11) を (4.8) に代入し μ^0, μ^2, ……, μ^{2j} の係数を比較すると

$$A=-\frac{1}{\beta}B_1+\frac{1}{3}H_0 \tag{4.12}$$

$$B_3-(1-\frac{1}{2\cdot3}f^2\beta)B_1+\frac{1}{3}\beta H_0=0 \tag{4.13}$$

……

となり $j>1$ に対しては一般に

$$B_{2j+1}-\left\{1-\frac{f^2\beta}{2j(2j+1)}\right\}B_{2j-1}-\frac{\beta}{2j(2j+1)}B_{2j-3}=0 \tag{4.14}$$

が得られる．そこで今

$$\frac{B_{2j+1}}{B_{2j-1}}=N_{j+1} \tag{4.15}$$

と置けば $j>1$ に対しては

$$N_{j+1}=1-\frac{\beta f^2}{2j(2j+1)}+\frac{\beta}{2j(2j+1)N_j} \tag{4.16}$$

となる．今 j が大きい場合を考えると，N_j は

$$N_j=-\frac{\beta}{2j(2j+1)}$$

の如くに次第に小さくなって0に近づくか，或はさうでないか，の2つの場合しかない．まづ後者の場合を考えると N は0に近づかないのであるから

$$N_{j+1}=1-\frac{\beta f^2}{2j(2j+1)} \tag{4.17}$$

であって $j\to\infty$ に於て N_j は1よりも小さい値から次第に極限値1に近づいて行く．故に (4.9) の右辺の或項から先の和は

$$M(\mu)(1+\mu^2+\mu^4+\cdots)=\frac{M(\mu)}{1-\mu^2}$$

の如き量よりも大きい．故に L と M とが共に μ の函数で然も $\mu=\pm1$ で∞にならないものとすると

$$\frac{1}{f^2-\mu^2}\frac{\partial\zeta'}{\partial\mu}=L+\frac{M}{1-\mu^2}$$

となり速度成分 u は

$$\begin{aligned} u&=\frac{i\sigma g}{\sigma^2-4\omega^2\cos^2\theta}\cdot\frac{\partial\zeta'}{a\partial\theta}\\ &=-\frac{i\sigma g}{4\omega^2a}\cdot\frac{(1-\mu^2)^{\frac{1}{2}}}{\mu^2-f^2}\cdot\frac{\partial\zeta'}{\partial\mu}\\ &=-\frac{i\sigma g}{4\omega^2a^2}\left\{(1-\mu^2)^{\frac{1}{2}}L+(1-\mu^2)^{-\frac{1}{2}}M\right\}\end{aligned}$$

となって両極 ($\mu=\pm1$) で $u=\infty$ となる．これでは実用にならない．従ってこの仮定は不可である．

そこで j が大きくなると N_j は

$$N_j=-\frac{\beta}{2j(2j+1)}$$

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* Sir William Thomson, Note on the "Oscillations of the First Species" in Laplace's Theory of the Tides," Phil. Mag. (4) 1. 279 (1875) [Papers, iv 248].

** G. H. Darwin, On the Dynamical Theory of the Tides of Long Period, Proc. Roy. Soc. XLI 337-342 (1886) [Papers i 336].

の如くに極限値 0 に近づくとすると勿論級数 (4.9) は μ の総ての値に対して有限である．本問題の解として採用す可きものは即ちこの場合である．

そこで (4.16) を変形すれば

$$N_j=\frac{-\dfrac{\beta}{2j(2j+1)}}{1-\dfrac{\beta f^2}{2j(2j+1)}-N_{j+1}} \tag{4.18}$$

となる．次に j の代りに $j-1$ と置けば

$$N_{j-1}=\frac{-\dfrac{\beta}{(2j-2)(2j-1)}}{1-\dfrac{\beta f^2}{(2j-2)(2j-1)}-Nj} \tag{4.19}$$

となるから今充分大きい j に対し

$$N_j=-\frac{\beta}{2j(2j+1)} \tag{4.20}$$

であるとしてこれを (4.19) に代入すれば N_{j-1} が得られる．

この操作を順次繰返せば

$$N_j,\ N_{j-1},\ N_{j-2},\ \cdots\cdots,\ N_2,\ N_1,$$

等の数値が得られる．更に

$$-\frac{\beta}{2\cdot3}B_{-1}=\frac{1}{3}\beta H_0$$

即ち

$$B_{-1}=-2H_0 \tag{4.21}$$

の如き B_{-1} を仮定すれば (4.14) は $j=1$ にも適用されるからこれに依って N_1 を決定すると

$$B_1=N_1B_{-1}=-2N_1H_0,\quad A=+\left(\frac{1}{3}+\frac{2N_1}{\beta}\right)H_0$$

となって B_1, B_3, … は次の如く決定される．

$$A=+\left(\frac{1}{3}+\frac{N_1}{\beta}\right)H_0,$$

$$B_1=-2N_1H_0,\quad B_3=-2N_1N_2H_0,$$

$$B_5=-2N_1N_2N_3H_0,\ \cdots\cdots$$

これらを (4.10) に代入し $\bar{\zeta}=H_0(\mu^2-\frac{1}{3})$ であることを考慮すると

$$\zeta/H_0=+2N_1/\beta+(1-f^2N_1)\mu^2+N_1(1-f^2N_2)\mu^4-\cdots$$
$$+\frac{1}{j}N_1N_2\cdots N_{j-1}(1-f^2N_j)\mu^{2j}+\cdots \tag{4.22}$$

となって解が得られる．

長周期潮汐の中で週期の最も短いものは太陰半月潮 M_f でこれに対しては $f=0.0133$ である．Darwin の計算を掲げると $\beta=40$ 即ち

$h=7260$ 呎の海洋では

$$\zeta/H_0=-0.1520+1.0041\,\mu^2-1.5228\,\mu^4+1.2187\,\mu^6$$
$$-0.6099\,\mu^8+0.2089\,\mu^{10}-0.0519\,\mu^{12}+0.0098\,\mu^{14}$$
$$-0.0014\,\mu^{16}+0.0002\,\mu^{18}+\cdots \tag{4.23}$$

となる．両極即ち $\mu=\pm1$ では，平衡潮高は $+\frac{2}{3}H_0$ で赤道上 ($\mu=0$) では全しく $-\frac{1}{3}H_0$ である．今上式で夫々 $\mu=\pm1$ 及び $\mu=0$ と置いて両極及び赤道上の潮高を計算すると次の様になる．

両極： $\zeta=+H_0\times0.1037=+\frac{2}{3}H_0\times0.1556,$

赤道： $\zeta=-H_0\times0.1520=-\frac{1}{3}H_0\times0.4561.$

次に $\beta=10$，即ち海が4倍深く $h=29040$ 呎とすると

$$\zeta/H_0=-0.2363+1.0016\,\mu^2-0.5910\,\mu^4+0.1627\,\mu^6$$
$$-0.0258\,\mu^8+0.0026\,\mu^{10}-0.0002\,\mu^{12}+-\cdots \tag{4.24}$$

両極： $\zeta=+H_0\times0.3137=+\frac{2}{3}H_0\times0.471$

赤道： $\zeta=-H_0\times0.2363=-\frac{1}{3}H_0\times0.709$

となる．即ち浅い海洋では潮高は平衡潮汐に比して極めて小さいが海が段々深くなるにつれて動力学的の潮高は次第に平衡潮高に近づくのである．

5. 無限に長い週期を有つ潮汐の極限形

Darwin の計算に於て

$$f^2=0$$

と置いたものは無限に長い週期を有つ潮汐を与えるがこの極限形は解式から見られる通り $f^2=0.00133$ とした太陰半月潮と大差がない．H. Lamb は $\beta=40$ $\beta=10$ 及び $\beta=5$ に対してこれを計算しているがこれを次に掲げる．

$\beta=40$ では

$$\zeta/H_0=-0.1515+1.0000\,\mu^2-1.5153\,\mu^4+1.2120\,\mu^6$$
$$-0.6063\,\mu^8+0.2076\,\mu^{10}-0.0516\,\mu^{12}+0.0097\,\mu^{14}$$
$$-0.0018\,\mu^{16}+0.0002\,\mu^{18}-+\cdots \tag{5.1}$$

で平衡潮高との比較は次の様になる．

両極： $\zeta=+\frac{2}{3}H_0\times0.154$

赤道： $\zeta=-\frac{1}{3}H_0\times0.455$

また $\beta=10$ に対しては

$$\zeta/H_0=-0.2359+1.0000\,\mu^2-0.5898\,\mu^4+0.1623\,\mu^6$$
$$-0.0258\,\mu^8+0.0026\,\mu^{10}-0.0002\,\mu^{12}+-\cdots \tag{5.2}$$

で

両極： $\zeta=+\frac{2}{3}H_0\times0.470$

赤道： $\zeta=-\frac{1}{3}H_0\times0.709$

となる．更に $h=58080$ 呎に対する $\beta=5$ では

$$\zeta/H_0=-0.2723+1.0000\,\mu^2-0.3404\,\mu^4+0.0509\,\mu^6$$
$$-0.0043\,\mu^8+0.0004\,\mu^{10}-+\cdots \tag{5.3}$$

で

両極： $\zeta=+\frac{2}{3}H_0\times0.651$

赤道： $\zeta=-\frac{1}{3}H_0\times0.817$

となる．

同じ β に対しては $f^2=0.00133$ なる太陰半月潮でも $f^2=0$ なる無限に長い週期の潮汐でも殆ど変化ないことは ζ/H_0 の式を見れば明らかであろう．

6. 一日潮汐

次に運動の性質が経度に関係する場合は，Laplace の第二種の振動及び同第三種の振動である．これは夫々一日潮汐及び半日潮汐に相当する．この場合水面の変形は最早や極軸の周りに対称でない．今 $s\neq0$ とし，Ω, u, v, ζ, は何れも

$$e^{i(\sigma t+s\lambda+\varepsilon)}$$

に比例すると考えると t と λ とを消去した式は次の様になる.

$$u=\frac{ig\sigma}{4\omega^2a(f^2-\cos^2\theta)}\left\{\frac{\partial\zeta'}{\partial\theta}+\frac{s\zeta'}{f}\cot\theta\right\},\qquad(6.1)$$

$$v=-\frac{ig\sigma}{4\omega^2a(f^2-\cos^2\theta)}\left\{\frac{\cos\theta}{f}\cdot\frac{\partial\zeta'}{\partial\theta}+s\zeta'\operatorname{cosec}\theta\right\},\qquad(6.2)$$

$$i\sigma\zeta=-\frac{1}{a\sin\theta}\left\{\frac{\partial(hu\sin\theta)}{\partial\theta}+ishv\right\}.\qquad(6.3)$$

(6.1), (6.2), (6.3) から u, v を消去すると

$$\frac{\partial}{\sin\theta\partial\theta}\left\{\frac{h(\sin\theta\frac{\partial\zeta'}{\partial\theta}+\frac{s}{f}\cos\theta\zeta')}{f^2-\cos^2\theta}\right\}$$
$$-\frac{h}{f^2-\cos^2\theta}\left(\frac{s}{f}\cot\theta\frac{\partial\zeta'}{\partial\theta}+s^2\zeta'\operatorname{cosec}^2\theta\right)$$
$$+\frac{4\omega^2a^2}{g}\zeta'=-\frac{3\omega^2a^2}{g}\zeta\qquad(6.4)$$

となる.

一日潮汐では

$$s=1,\quad\bar\zeta=H_1\sin\theta\cos\theta\,e^{i(\sigma t+\lambda+\varepsilon_1)}$$

であり, 半日潮汐では

$$s=2,\quad\bar\zeta=H_2\sin^2\theta e^{i(\sigma t+2\lambda+\varepsilon_2)}$$

と置けば宜しい.

天体の固有運動を省略すれば一日潮汐に対しては大体

$$\sigma=\omega\qquad(6.5)$$

と置いても大差はない. そこで

$$s=1,\quad f=\frac{1}{2}$$

と置き且つ

$$\zeta'=C\sin\theta\cos\theta\qquad(6.6)$$

と置く. 但し C は常数である. この式を (6.1)(6.2) に代入すれば u, v は次式の様になる.

$$u=\frac{i\sigma g}{\omega^2a}C,\quad v=-\frac{\sigma g}{\omega^2a}C\cos\theta\qquad(6.7)$$

この両式を (6.3) に代入すれば

$$\zeta=\frac{gC}{\omega^2a^2}\frac{dh}{d\theta}$$

となるから $\zeta=\bar\zeta+\zeta'$ であることに注意すれば

$$\bar\zeta+C\sin\theta\cos\theta=\frac{gC}{\omega^2a^2}\cdot\frac{dh}{d\theta}\qquad(6.8)$$

となる. 然るに

$$\bar\zeta=H_1\sin\theta\cos\theta\,e^{i(\sigma t+\lambda+\varepsilon_1)}$$

であるから (6.8) は

$$(C+H_1)\sin\theta\cos\theta=\frac{gC}{\omega^2a^2}\frac{dh}{d\theta}$$

となる. 今水深 h を一様とすればこの式の右辺は $=0$ であるから

$$C=-H_1,\quad\zeta=0\qquad(6.9)$$

となる. そこで水深の式を更に拡張して

$$h=h_0(1-q\cos^2\theta)\qquad(6.10)$$

と置けば

$$(C+H_1)\sin\theta\cos\theta=\frac{gC}{\omega^2a^2}\cdot 2qh_0\sin\theta\cos\theta$$

となって

$$C=\frac{H_1}{1-2qh_0g/\omega^2a^2}\qquad(6.11)$$

となり一日潮汐に関する解は

$$u=+\frac{i\sigma g}{\omega^2a}\frac{H_1e^{i(\sigma t+\lambda+\varepsilon_1)}}{1-2qgh_0/\omega^2a^2},$$
$$v=-\frac{\sigma g}{\omega^2a}\cdot\frac{H_1\cos\theta e^{i(\sigma t+\lambda+\varepsilon_1)}}{1-2qgh_0/\omega^2a^2},$$
$$\zeta=-\frac{2qgh_0/\omega^2a^2}{1-2qgh_0/\omega^2a^2}H_1\sin\theta\cos\theta\,e^{i(\sigma t+\lambda+\varepsilon_1)}$$

となる. 最も興味のあることは深さが一様な場合即ち $q=0$ のときには

$$\zeta=0$$

となり一日潮汐が無いことである. にも拘らず u, v, は零でないから潮流は確実に存在する.

勿論これは正確に $\sigma=\omega$ の場合で, 実際には一日潮汐が絶無なわけではないが後で Hough の理論で示す様に, 一般に動力学的潮汐論では一日潮汐が小さいことが結論される.

7. 半日潮汐: 特殊の場合

半日潮汐では $s=2$ で σ は大体 2ω に等しい. これは Laplace の第三種の振動であって

$$\bar\zeta=H_2\sin^2\theta\,e^{i(\sigma t+2\lambda+\varepsilon_2)}\qquad(7.1)$$

となる. 前と同様, $\sigma=2\omega$, $f=1$ と置いてしまえば太陰半日潮汐の場合には若干の差があるが大体の傾向を見るには充分である.

かゝる場合に対し Laplace は水深 h が

$$h=h_0\sin^2\theta\qquad(7.2)$$

即ち両極で深さが $=0$ で赤道で最も深い様な海洋に対し簡単な解を得た. 即ち C を常数とし

$$\zeta'=\zeta-\bar\zeta=C\sin^2\theta$$

として前同様にして C を決定すると

$$\left.\begin{aligned}u&=-\frac{i\sigma g}{\omega^2a}\cdot\frac{H_2\cot\theta\cdot e^{i(\sigma t+2\lambda+\varepsilon_2)}}{1-2gh_0/\omega^2a^2},\\ v&=+\frac{\sigma g}{2\omega^2a}\cdot\frac{H_2(1+\cos^2\theta)/\sin\theta\cdot e^{i(\sigma t+2\lambda+\varepsilon_2)}}{1-2gh_0/\omega^2a^2}\end{aligned}\right\}\qquad(7.3)$$

$$\zeta=-\frac{2h_0g/\omega^2a^2}{1-2gh_0/\omega^2a^2}H_2\sin^2\theta\,e^{i(\sigma t+2\lambda+\varepsilon_2)}\qquad(7.4)$$

となる.

現実の海洋の水深では $2h_0g>\omega^2a^2$ であるから潮汐は反転, 即ち起潮力と逆の符号を有つ. また (7.3) では速度が両極で無限に大きいことになる. かゝる解は, 水深が零になる以上止むを得ないことであるが現実に適合する解とは申せない.

録　　事

1. 日本海洋学会昭和33年度秋季大会及び臨時総会

昭和33年11月10日から14日迄，名古屋大学理学部において秋季大会及び臨時総会が行はれた．次第は次の通りである．

(1) 研究発表

11月10日午前の部（1～5，座長速水頌一郎）

1. 本州南方及び東方海域の黒潮流帯の変動　福岡二郎（気象研）　2. 日本沿岸における気象潮の分布と季節変化　宇野木早苗（気象研）　3. 沿岸ステーション用波浪計MR—Mark Ⅰの試作と現場試験について　降簱常雄（気象研）　4. Cauchy—Poisson 波に対する考察　三好寿（東水大）　5. 北太平洋における気象変化と海況変化との関連　半沢正男（気象庁）

10時40分より臨時総会，別項記載

午後の部（6～10，座長吉田耕造）

6. 船上のスタトスコープ記象に現われる海面の稍長周期振動について，喜多豊一（水路部）　7. シアーの流れに生ずる表面波の安定性について（第2報）　富永政英（東京学芸大）　8. 潮流を含む水理模型実験の相似性について　○速水頌一郎（京大理）樋口明生（京大防災研）吉田幸三（京大理）　9. 風洞水槽による風浪の研究（第2報）　国司秀明（京大理）　10. リュツオホルム湾附近の海底地形と底質について　庄司大太郎，佐藤任弘（水路部）

午後の部（11～15　座長三宅泰雄）

11. 海洋におけるMoの分布　菅原健（名大理），○岡部史郎（福岡学芸大）　12. 海水中における酸化態の沃素について　菅原健，○寺田喜久雄（名大理）　13. 海上観測資料の一取扱い　阿部友三郎（気象研）（福岡二郎代読）　14. Syn-bubble—bursting fractionation について　菅原健（名大理）　15. 油壺験潮場内，固定点の変動量について　山口生知（山形大）

11月11日午前の部（16～19　座長山口生知）

16. オホーツック海の水塊に対する一考察．大内正夫（京都学芸大）　17. オホーック海南西部の海況について　杉浦次郎（函館海気），（安井善一代読）　18. 沿岸氷下の海水の塩分垂直分布について　鈴木義男（北大低温研）　19. 海氷の弾性係数の測定　田畑忠司（北大低温研）

午前の部（20～23　座長富永政英）

20. 津軽海峡西口における漂流瓶投入結果について（第3報）　川上喜代四（7管水路部）　21. 津浪の伝播に関する一考察—Tsunami record の Amplitude envelope の形と travel distance との関係を主として　吉田耕造（東大理）　22. 赤道海域一齊海流測定について（1958年 1～2月）（第2報）　二谷穎男，○吉田昭三（水路部）　23. 地衡流境界値を用いる赤道潜流の研究　日高孝次（東大理）

午後の部（24～30　座長菅原健）

24. 1958月1～2月における西太平洋赤道帯の営養塩分布について　長屋裕（水路部）　25. 液浸型シンチレーションカウンターによる海水の放射能測定　島野次夫（水路部）　26. ケープタウン～南極大陸間の海洋化学的研究　鳥居鉄也（南極統合本部）　27. 日本沿岸海水のセシウム137含量　山県登（群馬大工）　28. γ線スペクトロメーターによる海底土中の放射能の測定　堀定清（水路部）　29. 放射化分析による魚体中の稀土類元素の分析について（第1報）　亀田和久（日本原研）　30. ストロンチュムとカルシウムの海水から魚えの移行量　○檜山義夫（東大農），市川龍資（放医研）

午後の部（31～37　座長佐々木忠義）

31. Sea mount の構造についての一考察　齊藤行正（大阪市立大）　32. 江の島，鎌倉附近における底質分布と流れについて　小向良七（水路部）　33. 塩素量の地理的分布特性と珪藻量ならびに組成との関係について　上野福三（三重県立大）　34. 本邦東北水域における基礎生産　○西条八束（東京都立大），市村俊英（東京教育大），大和田守，朝岡治（気象庁）　35. 水中濁度計の試作　○半谷高久（東京都立大），山口次郎（村山電気KK）　36. 海洋における白金電極電位の分布について　日向野良治（水路部）

11月12日午前の部（37～39　座長松平康雄）

37. 遠洲灘沖の冷水域について　森安茂雄（神戸海気）　38. 伊豆近海の海況変動について　○渡辺隆三，梅田次昌（水路部）　39. 海水又は湖水内に沈められる小板上の照度　向井正幸（信洲大理）

午前の部（40～43　座長庄司大太郎）

40. 採取した海水による散乱光の角度分布の測定　佐々木忠義（東水大，科研），渡辺精一，大柴五八郎，○岡見登（科研）　41. 海中光の偏光について　佐々木忠義（東水大，科研），渡辺精一，大柴五八郎，○岡見登（科研）　42. 海洋観測のロボット化に関する研究（第1報）　佐々木忠義（東水大，科研），○渡辺精一，大柴五八郎，岡見登（科研）　43. 密度不連続面における流速分布と剪断応力について（層流の場合）　奥田節夫（岡山大理）

午後の部（44～48　座長齊藤行正）

44. 的矢湾における潮流現象について　佐藤忠勇（的矢湾養蠣研）　45. 海水比重の 0°C への更正について　福尾義昭（京大理）　46. 海面における気泡の破裂による海水滴の生成について（第3報），浮遊気泡の形状に関する考察　鳥羽良明（京大理）　47. 外洋性漁場の海洋学的構造　北野清光（北水研）　48. 赤道域海流測定について（IGY 1958年）　高橋淳雄（鹿児島大水）

(2) 特別講演

午後の部（1～3　座長檜山義夫）

1. ソ連における海洋研究施設について　三宅泰雄（東京教育大）　2. 深層大循環に関する新しい研究　日高孝次（東大理）　3. Arctic Sea Ice Conference の報告　田畑忠司（北大低温研）

(3) 懇親会

11月12日午後6時より名古屋大学医学部共済会館で懇親会を行った．大会及び臨時総会が無事終了したのを喜び集るもの45名，テーブルスピーチや陰し芸迄とび出し名残りをおしみ7時30分散会した．

(4)　エックスカーション

11月13日9時10分の列車にのり先ず津に到着　県立三重大学に立寄り，昼食をおえ，菅島に向う．ここにて臨海実験所等を見学し同夜は同所に宿泊　新鮮な海の幸に舌鼓みをうち菅原，三宅両先生のスライドを見て後ゆっくり休む．14日鳥羽にいたり解散．

最後にかくも盛大な秋季大会及び臨時総会を催された名古屋大学理学部　菅原健先生を始め各位，並びに県立三重大学の諸氏に厚く謝意を表する．

2.　臨時総会次第

11月10日午前10時40分より臨時総会が開れた．議長に菅原健氏が満場一致推薦された．議題は役員会呈出の会則改正案の審議である．現行会則では臨時総会の規定がないが出席者全員これを臨時総会とみなす事に賛成し直ちに審議に入る．始めに日高会長より役員会がこの改正案を呈出した経緯について説明があり，次で会則改正担当の宇田委員長が外国出張で不在の為宇野木委員が詳しい説明並びに答弁にあたつた．

(1)　質　疑

主なる質疑は次の通りである．(敬称略)

三宅泰雄　外国からの会員を名誉会員にしたらどうか．

佐々木忠義　名誉会員は適当な表現で外人もふくめられるようにしたらどうか．

吉田耕造　この問題で宇野木の説明を求め，結局いいまわしをかえることとなる．

三宅　賛助会員にも選挙権をあたえるべきだ．

佐々木　常任評議員が遠隔の地の者になつたらどうするか．

田畑忠司　推薦母体が推薦する場合人数の制限を考えるべきだ．

亀田和久　評議員の専門別の人数をきめたい．

佐々木　評議員の推薦の場合広く海洋学全般からの推薦が望ましいという精神を生かすべきだ．

三宅　常任評議員と地方評議員を区別するのはおかしい．

速水頌一郎　評議員の定数を増したらどうか．

佐々木　評議員選挙には専門，所属機関を明記してほしい．

速水　評議員と委員を全然別にしたらどうか．会計，編集は委員にしたら如何．

佐々木　総会成立人数の 1/10 は少なすぎる．

尚地方より函館在住の会員で海洋気象台の方から地区毎の評議員は地区毎で選挙する事．北海道の評議員定員を4名にしたいという要望がった．又委員及び総会の成立人数にも質問があった．

栗田(東海水研)から評議員全般の任務をはっきり成文化する必要があるという意見が出た．

評議員の定数及びその選出方法は総会時間内では仲々結論が出ないので議長は大会参加全員にはかりこの問題を除き他の条項は異議のないものとして．問題点はこの大会中に小委員会をもうけ案をつくる事を相談し満場一致可決した．

(2)　小委員会

小委員会は11月10日午后5時から7時30分迄ひらかれた．　出席者(順序不同)　菅原健，速水頌一郎，猿橋勝子，亀田和久，須藤英雄，宇野木早苗，三宅泰雄，吉田耕造，福岡二郎，庄司大太郎，田畑忠司，佐々木忠義の12名．菅原氏が議長になって議論を進める．

こゝでの議論は主に評議員の選出方法に関しておこなわれた．常任地方の別を行うのがおかしいとの議論から三宅から東京を一地域として地区別の定員をきめたらよく常任評議員に相当するものを幹事としたらよいとの意見が出た．推薦制についてもこまかく推薦の方法をきめたらよいという佐々木の意見が出たが実際に具体的にきめられないから第14条の海洋学各部門から広く選出される事が望ましいという精神を生かすという事に帰着した．以上の議論から下記に示すような，修正案が用意された．

(3)　11日の臨時総会

この修正案は11月11日午后1時より改めて開かれた臨時総会(議長菅原健)において可決された．尚，総会の成立には委任状をみとめるが審議事項はなるべく早く総会前に会員に連絡するよう努めるべきだとの意見が多かった．又委任状によらない出席会員の最低数を規定したらよいとの意見もあつたが，規定が余り複雑になるので会員の良識を信じて採らない事にした．幹事及び幹事会についての説明は会則本文から除き，すべて評議員会細則に記すべさという少数意見があつた．

(4)　臨時総会における決定事項

以下に述べる第何条というのは，既にお手許に届けてある役員会提出の会則改正案のものである．なお次の決定事項は骨子のみであつて，字句の細かい修正は役員会に一任する事になつた．

○第6条 4.　名誉会員　本会に対しとくに功労のあつた者(正会員に限らない)のうちから総会の決議を受けて推薦される．会費は免除．

○第13条　本会の役員として評議員40名をおく．そのうち会長1名，副会長2名，幹事17名を設ける．

○第14条　◦常任評議員，地方評議員の区別をとりやめ，選挙細則に定めた地区毎(8ブロック)の定数を全国の会員が選挙する．　◦会長，副会長，監査，幹事は評議員の互選にする．

○第17条　庶務，会計，編集，その他の事務は幹事が分担執行する．　第18条　削除

○第22条　役員の欠員が生じた場合には，その補充を行はない．

○第23条　委員を設けるに必要な事項の中に編集を加える．

○第29条　総会の成立に必要な出席者は，委任状を含め全会員の 1/5 以上とする．

第33条　原文の通り，大会委員は大会委員長が委嘱する．

選挙細則

○選挙権，被選挙権は団体会員を除く全ての会員（正会員，賛助会員，名誉会員）が有する．

○地区の区分及び評議員定数は次の通り（定数基準は大体会員数に比例するものとした．即ち会員数10名に対し評議員1名の割合で端数は4捨5入）．

地区	評議員定数	会員数
北海道	4	44
東　北	2	22
東　京	17	172

地区	評議員定数	会員数
関東	3	27
東海道	2	15
関西	7	73
北陸，山陰	1	10
九州	4	39
計	40	402

○会員は立候補および推薦候補となることが出来る．評議員会はこれらの候補をとりまとめ，更に適当な候補者を追加し，これらを一括して，候補者名簿を作製する．
○評議員の選出に際しては，海洋学各部門から広く選出される事が望ましい．しかし具体的な専門別定数の決定とその選出方法は次の機会にまわす．

評議員会細則

○常任評議員会を設け，幹事会を設ける．
幹事会は，会長，副会長，監査，幹事より構成される．

3. 例会記事

(1) 9月例会（9月9日　於気象庁）　1. 時期遅れの南極帰国談　庄司大太郎(水路部)　2. 台風5811号の高潮について　関重雄(気象庁)
(2)10月例会（10月14日　於東大地球物理）　最近の有孔虫研究　特に深海研究について　内尾高保(東大綜合研)

4. 役員会

(1)主なる決定事項．　9月の役員会で11月の臨時総会に呈出すべき会則改正案の最終審議を終了した．10月の役員会では11月の秋季大会の細目打合．
(2)主なる報告事項．　9月の役員会で日高会長より今年5月のIGYの一環としての赤道海域一齊海流調査の際さつま，拓洋の2隻が異常な人工放射能の影響をうけた事に対し，核爆発の行はれる間は協力出来ない旨，地研連の海洋分科会としてアメリカに警告を発したという説明があった．尚鹿児島大の敏天丸，水産大の海鷹丸は各々観測計画を変更した．この中海鷹丸は10月中にニュージーランドに向け出発するが，その際　深層流の観測を数点行う．拓洋も今後予算があれば赤道海域でもう一度観測を行う由である．

5. 会員消息

(1)本会評議員大久保明氏（気象庁）は原子力局の留学生として9月 Johns Hopkins 大学へ約1年間の勉学の予定で出発した．同氏は Chesapeake Bay Institute で研究される由．
(2)会員小野延雄氏(北大低温研）は南極観測隊の海洋観測員として11月12日東京港を出発した．御健闘を祈る．
（以上1～5項までの文責，福岡二郎）

6. ユネスコ，クーポン制度のお報せ

この制度は大変便利なものですがまだ充分御存知のない方もあるのでお報せします．
1. ユネスコ，クーポンで購入出来るもの
(a) 出版物，書籍，雑誌の予約購読，マイクロフィルム複写物，地図，学会々費等．(b) 映画フィルム　研究用フィルムの上映プリント，複写プリント等．　(c) 科学資料　i 光学器械　ii 秤及び衡器　iii 実験室用ガラス器　iv 電気測定器　v 分析用及び臨床用器具　vi 送風装置，真空ポンプ等　vii ラジオ部品及び電気器具　vii 温度計，恒温装置　ix 小工場用手動器具等　x 気象器械，測量器，工学用器械　xi 純粋化学品，培養基等　xii 海図，模型，スライド等　〔但し(a)は購入額に制限はないが，(b),(c)は一品目 $2000 迄のものに限る．〕
2. このクーポンは日本学術振興会（東京都台東区上野公園内　日本学術会議内，）Tel. (82) 4522, 4523 で取扱っている．同会内ユネスコ課に詳しい事をお問合せ下さい．
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7. 在外外国人，在外日本人会員の会費について

昭和34年度より(**昭和34年4月1日より**)，外国人会員で目下外国にあるもの，及び日本人会員で目下外国にあり，雑誌を自分が現在いるところに送ってもらいたいものの会費を**米貨2ドル50セント**（邦価900円に改訂いたしますから御承知下さい．
外国人会員で日本に滞在中のもの及び在外日本人会員で雑誌を留守宅に送ってもらいたいものの会費は従来通り600円です．

8. 編集記事

(1)日高孝次博士の特別寄稿“潮汐理論”は1号置きに掲載し，後2回で終了の予定．
(2)第15巻第1号の手持原稿は3篇です．

9. 国際海洋会議論文提出者へお知らせ

さきに本誌 (Vol. 14, No. 2) 上でお知らせした国際海洋会議 (International Oceanographic Congress) へ論文を提出される方はアブストラクトのコピーを同時に，

東京都台東区上野公園内
日本学術会議内
海洋学研究連絡委員会委員長

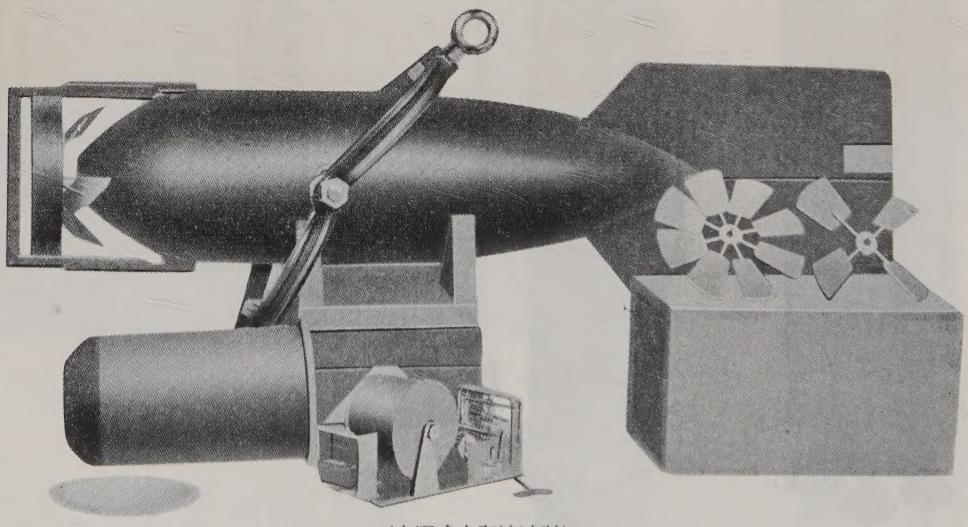
あて送附されるよう同委員長から衆知方連絡がありました。(2月1日迄必着)

10. First International Conference on Waste Disposal in the Marine Environment 開催のお知らせ

表記の会議が今夏米国加州バークリーで開かれます。くわしい事は，

Department of Conference
Univeritsy Extension
University of California
Barkeley 4, Calif., U. S. A.

にお問合わせ下さい。



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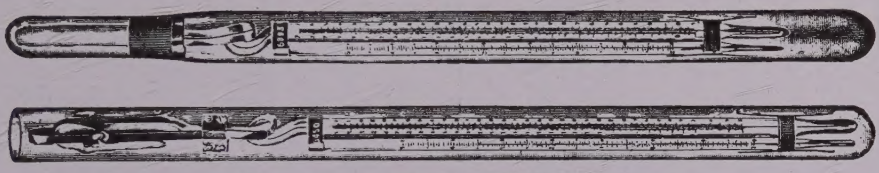

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